A Closer Look at Vertical Antennas With Elevated Ground Systems

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INTRODUCTION

Among amateurs there's been a long running discussion regarding the effectiveness of a vertical with an elevated ground system compared to one using a large number of radials either buried or lying on the ground surface. NEC modeling has indicated that an antenna with four elevated λ/4-radials would be as efficient as one with 60 or more λ/4 ground based radials. Over the years there have been a number of attempts to confirm or refute the NEC prediction experimentally with mixed results. These conflicting results prompted me to conduct a series of experiments directly comparing verticals with the two types of ground systems. The results of my experiments were reported in a series of QEX[1-7] and QST[8] articles (.pdf files of these articles are posted at: www.antennasbyn6lf.com). From these experiments I concluded that at least under ideal conditions four elevated λ/4 radials could be equivalent to a large number of radials on the ground.

Confirmation of the NEC predictions was very satisfying but that work must not be taken uncritically! My articles on that work failed to emphasize how prone to asymmetric radial currents and degraded performance the 4-radial elevated system is. You cannot just throw up any four radials and get the expected results. I'm by no means the first to point out that the performance of a vertical with only a few radials is sensitive to even modest asymmetries in the radial fan[10,11, 12], the presence of nearby conductors or even variations in the soil under the fan[9]. These can cause significant changes in the resonant frequency, the feedpoint impedance, the radiation pattern and radiation efficiency. These problems have been pointed out before but as far as I can tell no detailed follow-up has been published. Besides the practical problem of construction asymmetries, at many QTH's it's simply not possible to build an ideal elevated system even if you wanted to. There may not be enough space or there may be obstacles preventing the placement of radials in some areas or other limitations. I think it's very possible that some of the conflicting results from earlier experiments may well have been due to pattern distortion and increased ground loss that the simple 4-wire elevated system is susceptible to.

As the sensitivity of the 4-radial system and it's consequences sunk into my consciousness I began to strongly recommend that people use at least 10-12 or more radials in elevated systems. Although I have heard anecdotal accounts of significant improvements in antenna performance when the radial numbers were increased to 12 or more, I have not seen any detailed justification for that. What follows is my justification for my current advice.

My original intention for this article was to illustrate the problems introduced by radial fan asymmetries and discuss some possible remedies. But in the process I came to realize that before going into the effects and cures for asymmetries it was necessary to first understand the behavior of ideal systems. Ideal systems can show us when and why they are sensitive and point the way towards possible cures or at least ways minimize problems. The discussion of ideal antennas (over real ground however!) also illustrates a number of subtleties in the design and possibly useful variations that differ somewhat from current conventions.

For these reasons, after some historical examples of elevated wire ground systems, I'll spend a lot of time analyzing ideal systems and then move on to the original purpose of this article that was asymmetric radial currents and how to avoid them. At the end of this article I summarize my advice for verticals using elevated ground systems. While much of what follows is derived from NEC modeling, I have incorporated as much experimental data as I could find and compared it to the NEC predictions to see if NEC corresponds to reality.

Prior work on elevated ground systems

Looking through my files I realized I had a lot of information on elevated ground systems: Moxon[11, 12], Shanney[13], Laport[14], Doty, Frey and Mills[9], Weber[10], Burke and Miller[15, 16], Christman[18-33], Belrose[39, 42] and many others. I also found a good deal of my own work, some published but most not, so I decided to pull it all together and add some recent modeling work to shed light on the behavior and the causes and possible cures of some of the problems associated with elevated radial systems. Please note that there is an extensive list of references at the end this article.

Some history

In the early days of radio, operating wavelengths were in the hundreds or thousands of meters. Ground systems with λ/4 radials were rarely practical but very early it was recognized that an elevated system called a "counterpoise" or "capacitive ground", with dimensions significantly smaller than λ/4, could be quite efficient. Figure 1 shows a typical example.

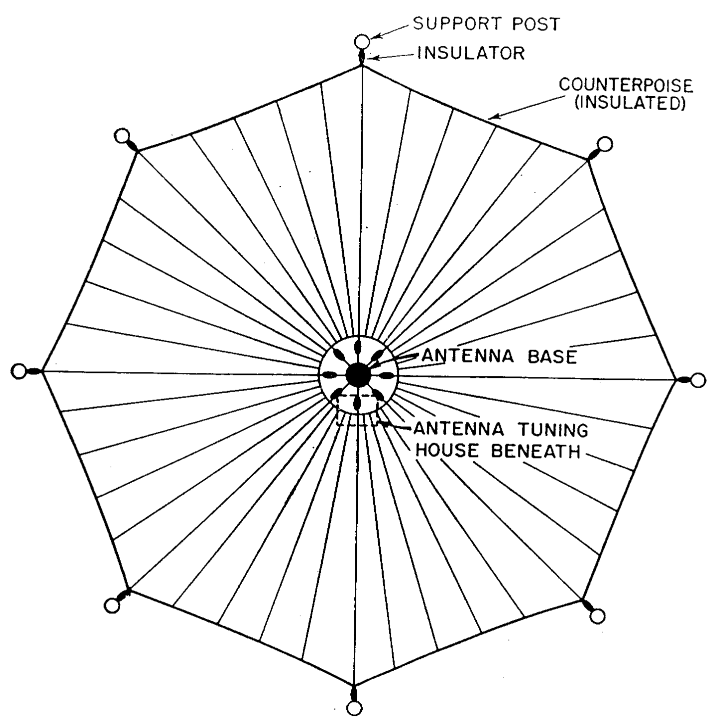

Figure 1 - A typical counterpoise ground system. Figure from Laport[14].

Here is an interesting quotation from Laport[14] regarding counterpoises:

"From the earliest days of radio the merits of the counterpoise as a low-loss ground system have been recognized because of the way in that the current densities in the ground are more or less uniformly distributed over the area of the counterpoise. It is inconvenient structurally to use very extensive counterpoise systems, and this is the principle reason that has limited their application. The size of the counterpoise depends upon the frequency. It should have sufficient capacitance to have a relatively low reactance at the working frequency so as to minimize the counterpoise potentials with respect to ground. The potential existing on the counterpoise may be a physical hazard that may also be objectionable."

Laport was referring to counterpoises that were smaller than λ/4 in radius. In situations where λ/4 elevated radials are not possible amateurs may be able to use a counterpoise instead. Unfortunately, beyond the brief remarks made here, I have to defer further discussion of counterpoises to a subsequent article even though I think they may be very useful to amateurs in some situations.

Rectangular counterpoises, some with a coarse rectangular mesh, were also common. A rather grand radial-wire counterpoise is shown in Figure 2.

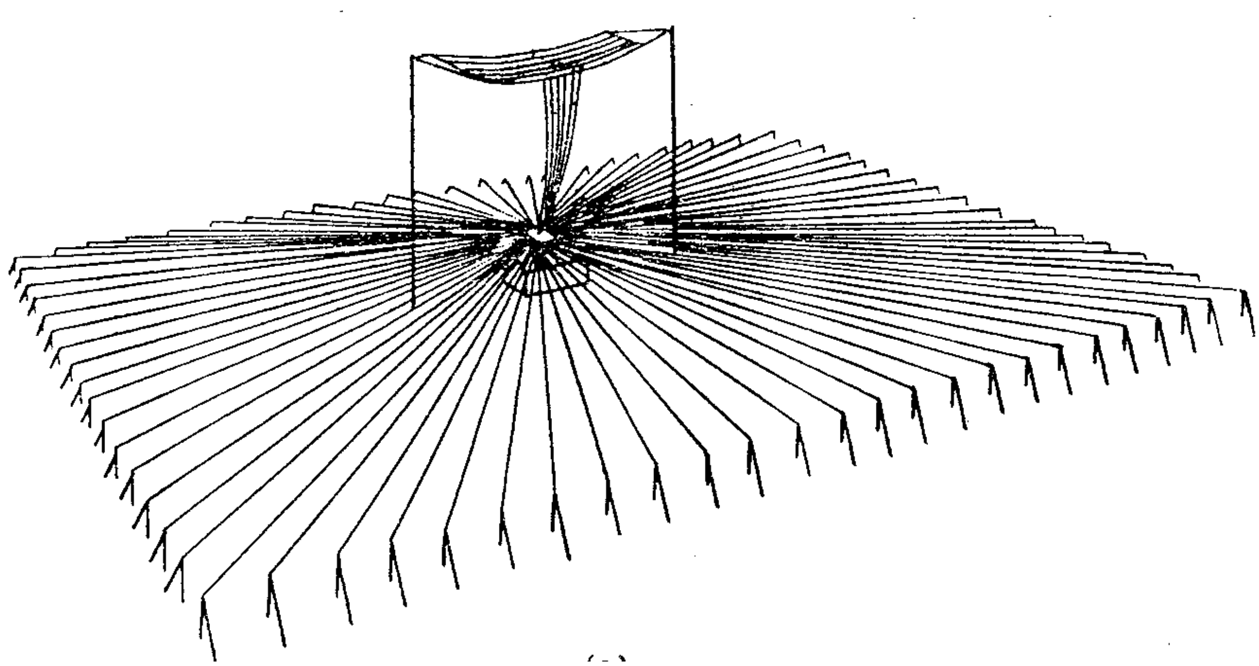

Figure 2 - A very large LF elevated ground system. From Admiralty Handbook of Wireless Telegraphy, 1932 [34].

Amateurs also used counterpoises. Figure 3 is a sketch of the antenna used for the initial transatlantic tests by amateurs (1BCG) in 1921-22[35, 36]. The operating frequency for the tests was about 1.3 MHz (230m). At 1.3 MHz λ/4 = 189' so the 60' radius of the counterpoise corresponds to ≈0.08λ.

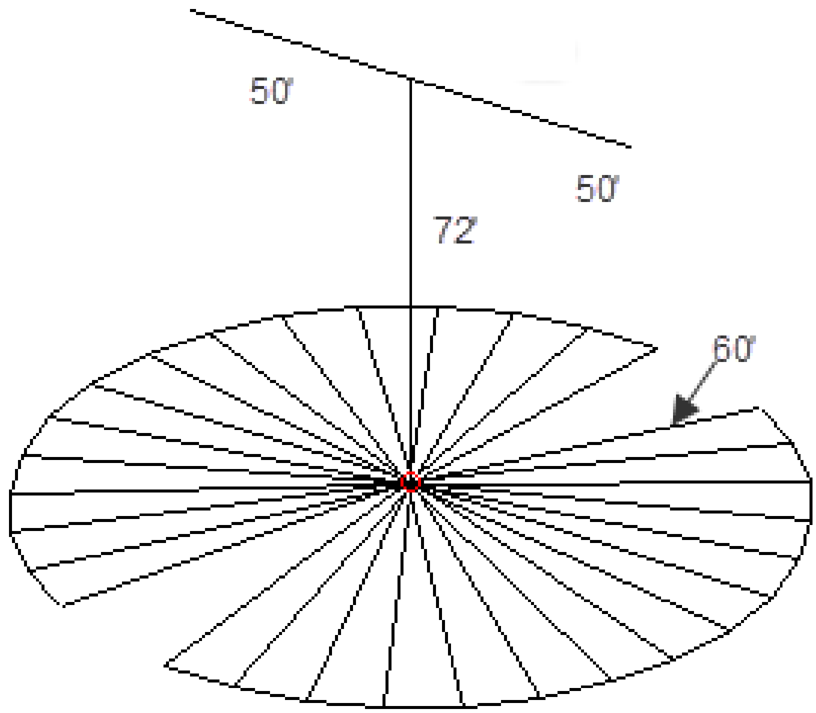

Figure 3 - EZNEC model of the 1BCG antenna.

Note that in all these examples a large number of radials are used. The use of only a few radials, initially with VHF antennas elevated well above ground, seems to have started with the work of Ponte[37] and Brown[38].

BEHAVIOR WITH IDEAL RADIAL FANS

In this section we'll look at verticals with a length (H) ≈ $\lambda_o/4$ (λ_o is the free space wavelength) and symmetric elevated radial systems where the height above ground (J) and the number (N) and length (L) of the radials is varied. We'll also look at the effect of soils with different characteristics from poor to very good. Even though we will be looking at verticals with H ≈ $\lambda_o/4$, keep in mind that elevated ground systems can also be used with verticals of other lengths, with or without loading, inverted L's, etc. There can also be multi-band operation.

NEC modeling

Figure 4 shows a typical model of a vertical with a radial system. Except as noted, the following discussion will focus on operation on 3.5-3.8 or 7.0-7.3 MHz as the operating band and 3.65 or 7.2 MHz as a spot frequency near mid-band. The conductors (both the vertical and the radials) are lossless #12 wire. Most of the modeling was done over real grounds. The modeling used EZNEC Pro4 v.5.0.45, using the NEC4D engine. The use of NEC4D over real soils gives the correct interaction between ground and the antenna. Excellent free programs based on NEC2 are available[41] but these do not properly model the ground-antenna interaction so that results obtained from them must be used with some caution. For HF verticals close to ground this is an important limitation.

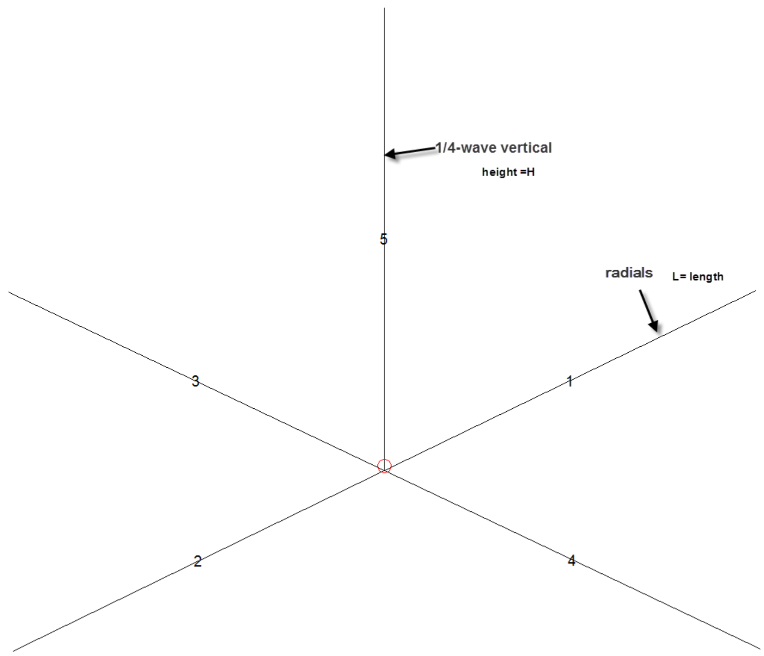

Figure 4 - λ/4 ground-plane vertical with four radials.

The effect of element dimensions on performance

The simplest idea of a ground-plane antenna is that you take a quarter-wave vertical and add four quarter-wave radials at the base. It is well known that the elements of a dipole will be a few percent shorter than λ_o so it is usually assumed that in a ground-plane antenna the vertical and the radial lengths will also be a few percent less than λ_o. Typically it is assumed that the vertical and the radials will be <u>individually</u> resonant at the operating frequency. Unfortunately it's not that simple because the vertical is coupled to the radials and both interact strongly with ground because, at least at lower HF (<20m), the base of the vertical and radial fan will usually be only a fraction of λ_o above ground. What you have in reality is a coupled multi-tuned system with complicated interactions. It turns out that there are a wide range of pairs of values for H and L that result in resonance or Xin = 0 at the feedpoint (where Zin = Rin + j Xin). Some of these combinations where neither the vertical nor the radials are individually resonant may be useful.

<u>Antenna resonance and element dimensions</u>

The free space wavelength (λ_o) at a given frequency in MHz (f_{MHz}) is:

$$\lambda_o = \frac{299.792}{f_{MHz}}\ [m] = \frac{983.570}{f_{MHz}}\ [feet]$$

At 3.65 MHz, $\lambda_o/4$ = 67.368'. If we model a resonant λ/4 vertical over perfect ground using #12 wire we find that at 3.65 MHz λ/4 = H=65.663' that is about 3.5% shorter than $\lambda_o/4$.

To take into account the effect of ground on radial resonance for a given value of J and soil characteristic, it has been suggested that we can erect a low dipole at the desired radial height (J) and trim its length to resonance. An example of this is given in Figure 5.

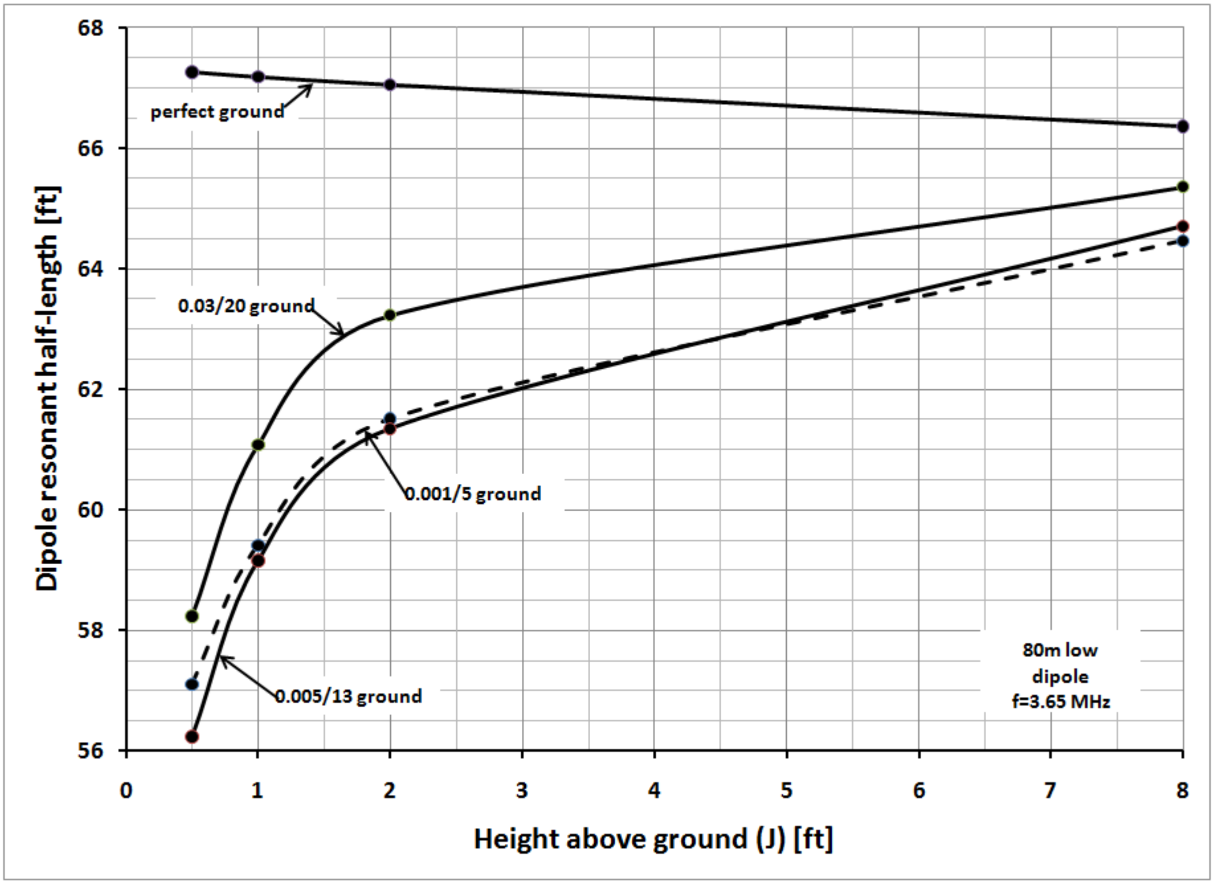

Figure 5 - Dipole half-length for resonance for different values of J and different soils.

For J=8', depending on the soil, L varies from 64.5' to 66.4'. As we reduce J we find that L gets smaller. The shift in resonance for radials close to ground has also been demonstrated experimentally[2]. Figure 6 shows the measured radial current at 7.2 MHz on 33' radials (sum of four radials). Clearly this radial is λ/4 resonant at a lower frequency than 7.2 MHz! As Figure 5 predicts the effect gets much larger for small values of J.

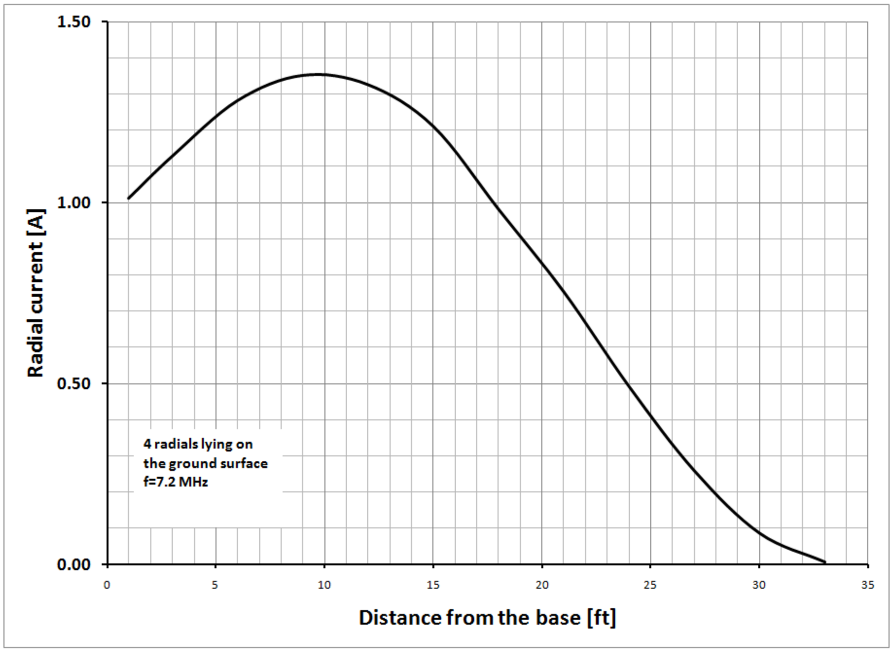

Figure 6 - Measured current on a 33' radial at 7.2 MHz. Four radials lying on the ground surface.

What do we mean by "resonant' values for H and L independently? It's not just that the reactances cancel at the feedpoint. When I say "the resonant length for H or L" I'm talking about the case where the current distribution on the vertical and the radials independently corresponds to resonance: i.e. the current just reaches a maximum at either the base of the vertical or at the inner ends of the radials. If either H or L is made longer than resonance, the current maximum will move out onto the radials or up the vertical. Figure 7 shows the current distribution on a vertical and the radials for three combinations of H and L each of which yield Xin = 0 at the feedpoint.

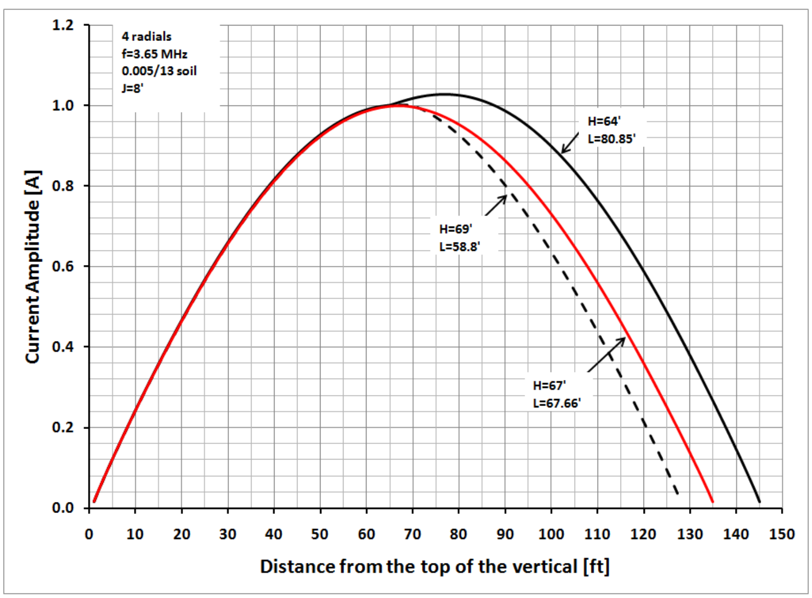

Figure 7 - Current distribution on the vertical and the radials. The current starts at the top of the vertical, runs to the base and then out along the radials. The radial current is the sum of the currents in the four radials. The currents are for 1 A_{rms} at the feedpoint.

To better understand what's happening we can expand Figure 7 around the 1 A feedpoint (indicated by the arrow) as shown in Figure 8.

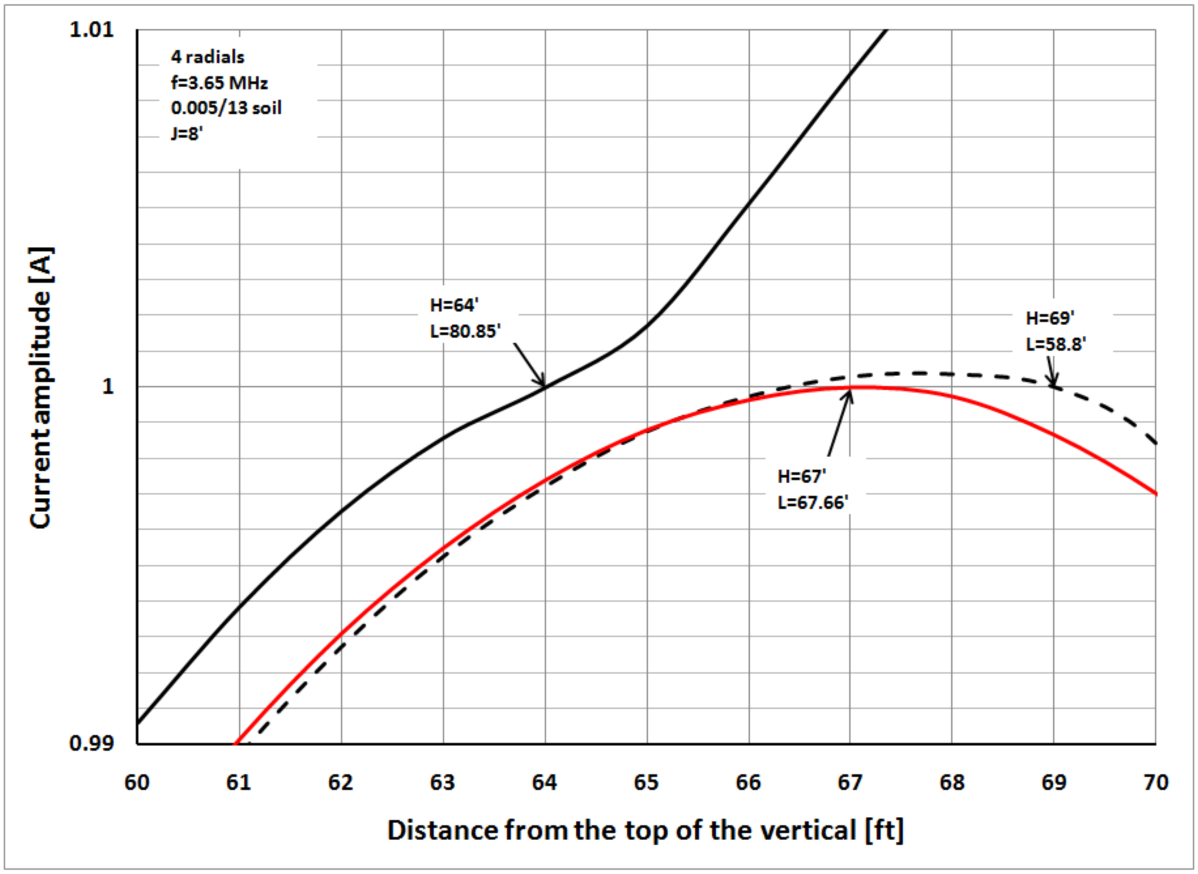

Figure 8 - Current distribution on the vertical and the radials expanded around the feedpoint. The arrows point to the junctions between the vertical and the radials.

For H=64' and L=80.85', the current on the vertical has not peaked so the vertical is too short for resonance. However, the radial current peak is well out on the radials so clearly the radials are too long for resonance. The reactance of the vertical and the radials cancels at the feedpoint so the antenna is "resonant" but not the vertical and radials individually. Similarly, for H=69' and L=58.8', the current in the vertical peaks and begins to fall (moving from the top to the bottom of the vertical) before the feedpoint is reached. Again, we have a resonant antenna but the vertical and the radials are not individually resonant. However, if we set H=67' and L=67.66', both the vertical and the radials are λ/4 resonant individually.

The "resonant" (by the definition given above) length of the vertical is 67' and the "resonant" length for the radials is 67.7', both of these lengths are substantially different than the value we got earlier for λ/4 resonance for a vertical over an infinite perfect ground-plane (65.7'). The "resonant" radial length of 67.7' is quite different from the dipole 8' over average ground (64.7'). H and L are actually closest to λ_o (67.4'). What we have just seen is only one particular example. If we change J and/or the soil characteristics and/or the number of radials these lengths will change!

Setting up the antenna so that both the vertical and the radials are individually resonant turns out to not be so simple and we might ask, "is it really necessary to have both the vertical and the radials resonant individually?" It turns out that there are other considerations besides the current distribution with regard to the choice of L for a given H. It is possible to use values of L where Xin ≠ 0 and compensate for that with a tuning impedance at the feedpoint for example or perhaps use some top-hat loading. In addition, in some situations it may not be possible to have radials long enough to make Xin = 0 while keeping the radial fan symmetric. Further, it has been suggested[10] that radials with L <λ/4 or >λ/4 are a possible cure for radial current division inequality. So we have reasons to investigate the effect of variations in vertical height and radial length on antenna behavior.

For each value of H, number of radials (N), height above ground (J), ground characteristic (σ and ε_r) and choice of operating frequency, there will be some radial length (Lr) that makes the antenna resonant (i.e. Zin = Rin + j Xin, where Xin = 0 at resonance). That's a lot of variables! So we will just look at a few examples to get a general idea of what happens.

Figure 9 gives an example of the variation in the value for L that results in resonance at the feedpoint (Lr) (Xin=0) as a function of N and several values of H, with fixed values of f, J and soil.

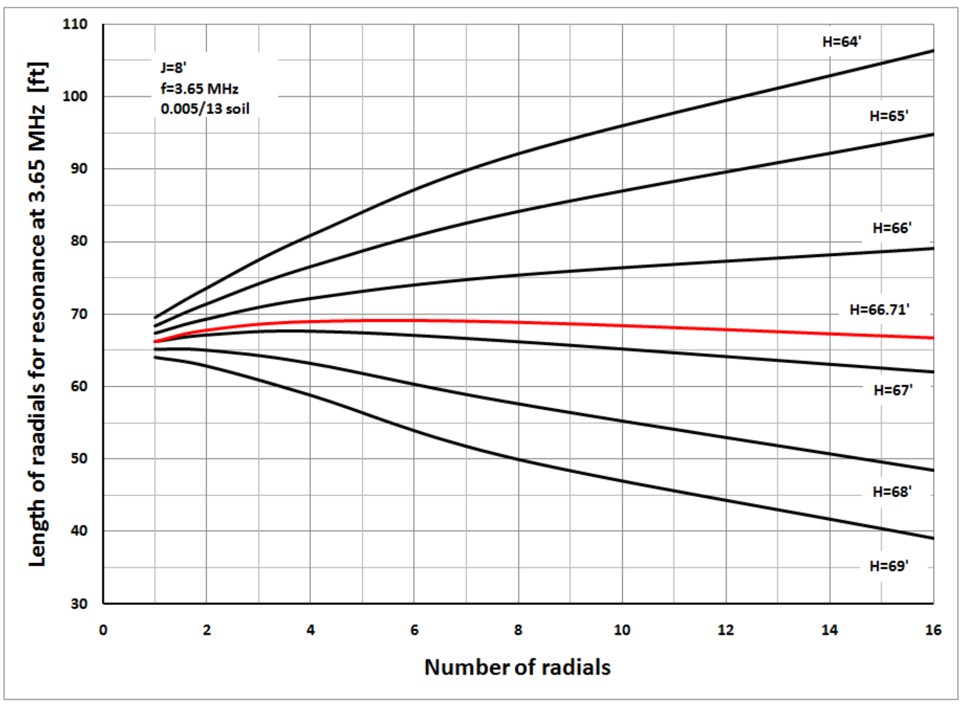

Figure 9 - Examples of the effect of radial number on the radial length for resonance at 3.650 MHz (Lr) for several different values of H.

Notice how widely Lr varies with N for most values of H although there is one value for H (66.71') that seems to have only a small variation in Lr as N is changed. Note also how much shorter Lr becomes when H is increased by a few feet. This could be very useful in situations where space for the radial fan is limited. On the other hand note how quickly Lr grows when H is shortened. For N=16 we see that when H=64', Lr=106' but for H=69', Lr is only 39'! That's a difference in Lr of almost 3:1. If you cannot make H long enough all is not lost! A bit of top loading can accomplish a similar effect as increasing H.

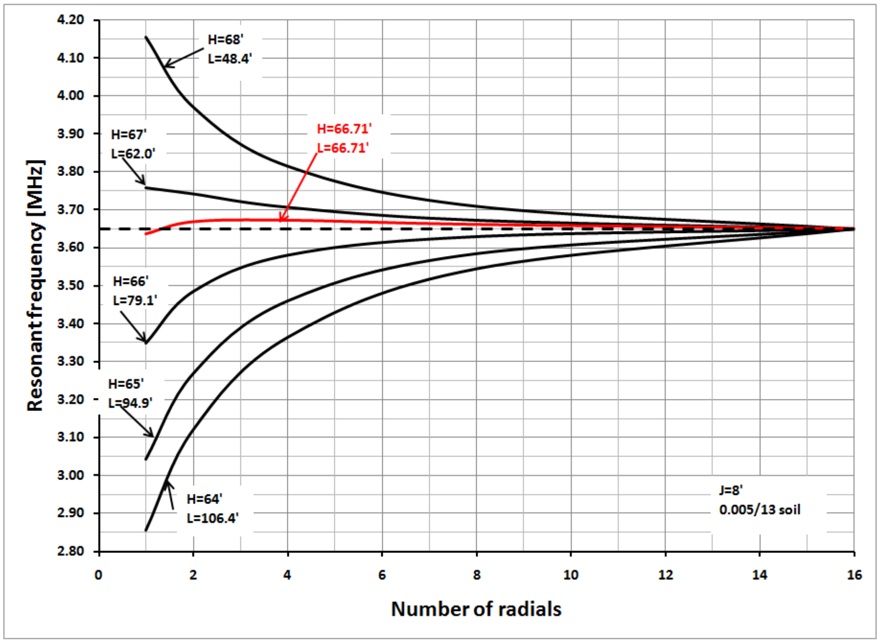

Figure 10 - Resonant frequency of the antenna as a function of radial number for several combinations of H and L that are resonant at 3.650 MHz with N=16.

Another way to explore the interaction between L and N is to set L equal to Lr for some value of N (say 16 radials) and while watching the resonant frequency (fr), vary the number of radials as shown in Figure 10. Note that the most stable fr is where H=L=66.71' that is relatively close to the values we got earlier for independently resonant vertical and radials (be careful, this is particular to this example; things will vary with different J, ground type, etc). Note that for H a bit tall fr decreases as radials are added but if H is a bit short fr goes increases as radials are added. This kind of behavior can be confusing if you are trimming the radials to resonate at a particular frequency, especially if you add some radials. It is possible you could add some radials and then have to make all the original radials longer!

This raises the question: do real antennas actually behave this way? During the ground system experiments I saw exactly this kind of behavior. For the 160m vertical fr went down as I added radials but for the 40m verticals fr went up with radial number. Figure 11 shows graphs of experimental measurements, one for 160m and the other for 40m. Real antennas do behave as the modeling predicts.

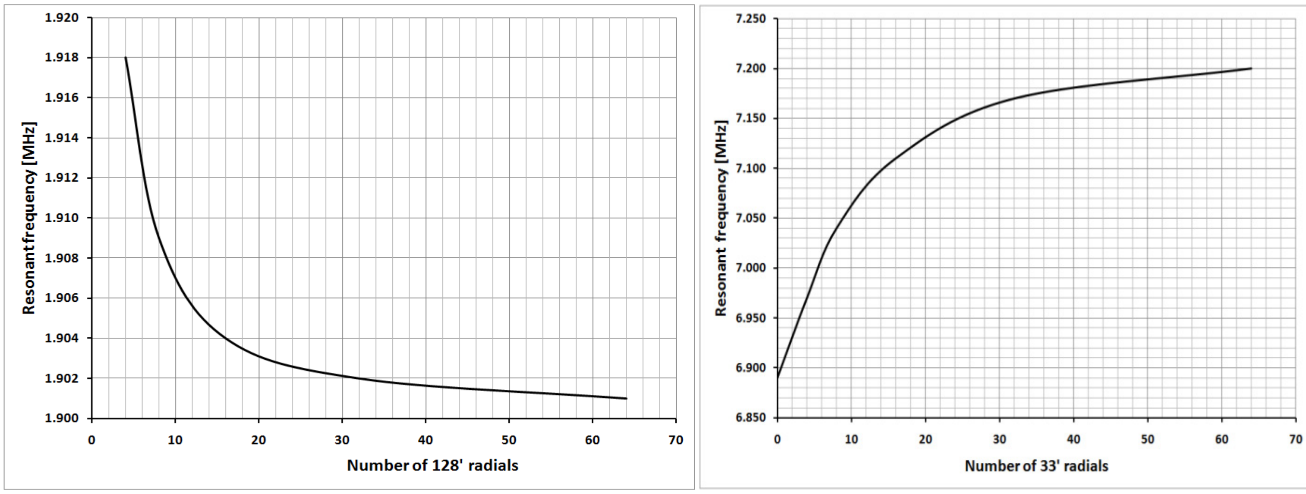

Figure 11 - Experimental measurements of the effect of radial number on resonant frequency.

At this point it's pretty clear that there is considerable interaction between the variables (H, L, J, etc) but it is not obvious yet if there are optimum combinations (some better than others).

The effect of radial length on efficiency

It turns out that the values for both N and L can have a significant effect on the efficiency of the antenna. Burke and Miller[15] published a very interesting paper in 1989 with the results of NEC modeling of both elevated and buried radial systems for a wide range of N, L, J and soil characteristics. I read this paper many years ago but I have to admit that it did not dawn on me just how much important information was there. Recently the light dawned as I re-read the paper and some additional graphs that Jerry Burke kindly sent me, so I have been redoing some of their modeling. Some of the Burke-Miller graphs were plots of average gain (Ga) versus radial length with radial number as a parameter. Ga is a useful proxy for radiation efficiency in that it gives the proportion of the input power to the antenna that is actually radiated into space. Ga is the ratio of the radiated power (Pr) to the input power (Pin) in dB (Ga=10 Log [Pr/Pin]). All of the power dissipated in the earth, including the near-field losses and reflections in the far-field, are subtracted from the input power. What is actually done is to integrate the power flow across a hemisphere with a very large radius centered on the antenna. The total power flowing through the surface of the hemisphere is Pr. I should emphasize that this is the power radiated towards the ionosphere, power in the ground-wave is considered a loss. For amateurs where sky-wave propagation is the norm at HF this makes sense.

The Burke-Miller graphs used a constant value for H. I will begin with similar graphs but for amateurs it is more likely that as L is increased H will be decreased to maintain resonance at a given frequency, so I will also show that style of graph.

Figure 12 is an example of the effect of radial length and radial number on Ga of the antenna when H is kept constant (68' in this example).

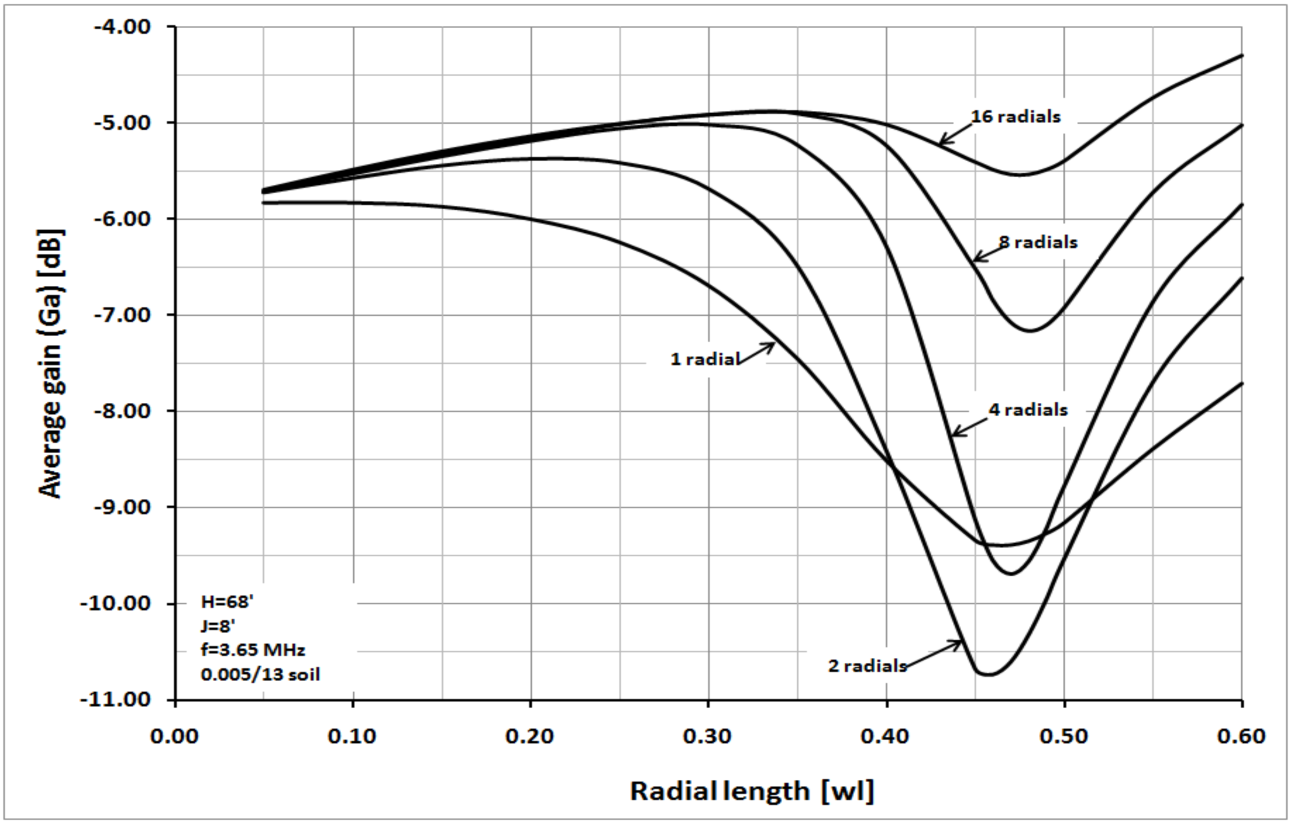

Figure 12 - Average gain as a function of radial length (in wavelengths, λ_o) and number of radials.
H=68', J=8', f=3.650 MHz and 0.005/13 soil.

Figure 12 has some interesting features:

1) Beginning with short values for L, Ga increases slowly up to a maximum. Below this point using radials somewhat shorter than λ/4 does not seriously reduce the efficiency.

2) Above the maximum however, there is a large dip! The bottom of the dip can be as much as -7 dB before Ga rises again for longer lengths.

3) Up to the length where Ga starts to fall, increasing N doesn't make much difference in Ga as long as you have four or more radials but increasing N does push the dip towards longer radial lengths and reduces the depth of the dip.

Figure 12 is for the case where J=8'. If we reduce J the Ga graphs will change as illustrated in Figure 13.

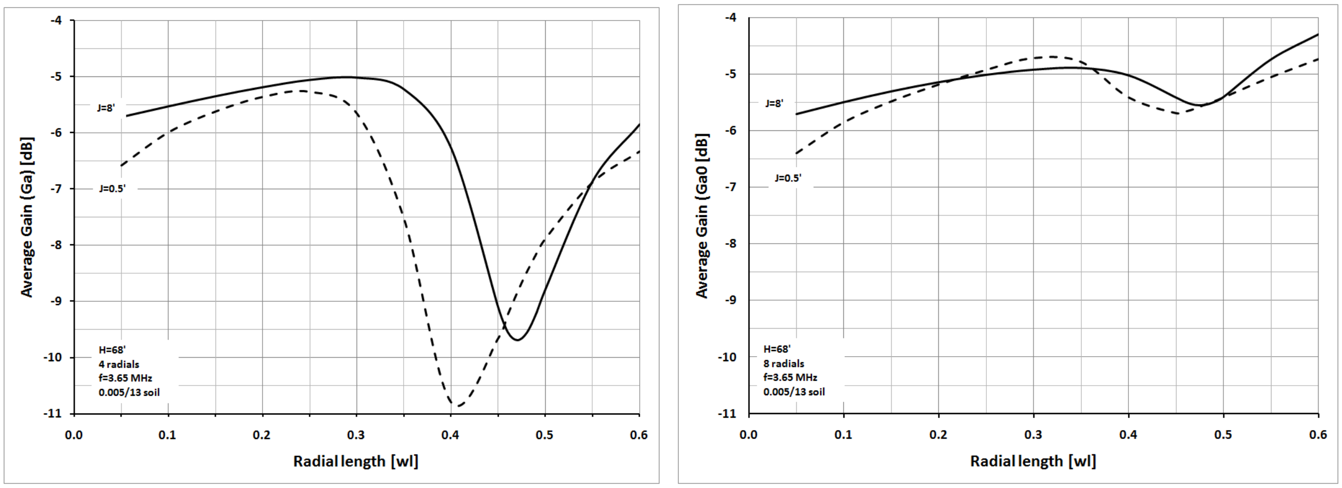

Figure 13 - Comparison of the effect on Ga for J=8' & 0.5'. N=4 & 8 and L is in λ_o.

As the antenna is moved closer to ground the efficiency starts to fall and the dip gets deeper and starts at shorter values for L. In fact if you push J down to 1" or less (i.e. radials lying on the ground surface) the notch gets even deeper and begins to fall off at lengths well below $\lambda_o/4$. Note however, that the effect is substantially reduced when larger numbers of radials are used.

One of the suggestions for improving current division between radials[10] was to make them substantially longer than $\lambda_o/4$, i.e. $L=3\lambda_o/8$. As Figures 12 and 13 show, that's probably not a good idea unless you're using 16 or more radials but with that many radials current division will already be much improved as we'll see shortly. Before getting carried away with conclusions we have to ask: do real antennas actually behave this way and do we have any experimental verification? As part of the ground system experiments reported in QEX[1-7] and QST[8], I measured the signal strength as N and L were varied with H constant. Figure 14 is a typical result.

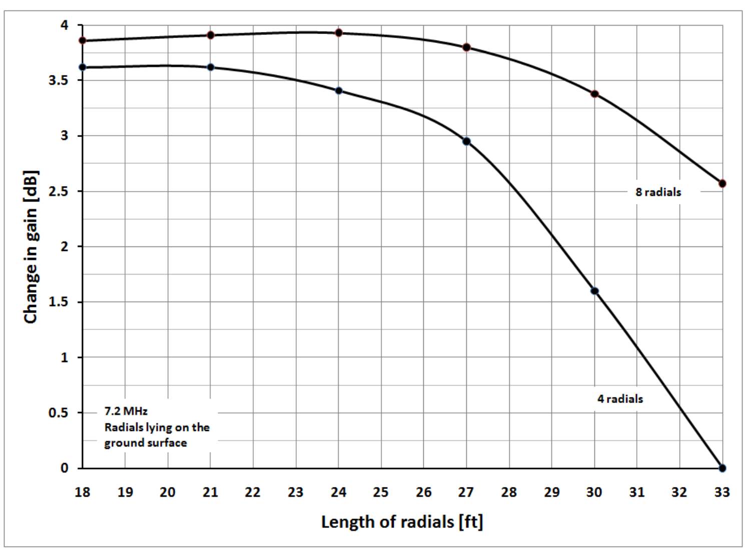

Figure 14 - Far-field change in signal strength as L and N are varied. Radials are lying on the ground surface. f= 7.2 MHz.

I have to admit that during the experiments I did not make the connection between my measurements and the work of Burke and Miller[15] so I only extended the radial lengths out to slightly less than $\lambda_o/4$. But we can still see the predicted behavior:

1) for L short, the gain rises slowly to a point where it starts to fall.

2) When L is large the dip in gain is large.

3) Increasing N reduces the dip and moves it to larger values for L.

Besides the data shown in Figure 14, I ran spot checks on the gain with sixteen and thirty two 33' radials. These were also in agreement with the NEC predictions. I think it's pretty clear that NEC is telling us the truth and we need to pay attention! Radial length is an important consideration.

Figures 12-13 are for σ=0.005 and ε_r = 13, Figure 15 shows the effect of different soil characteristics on Ga for given H, J and N.

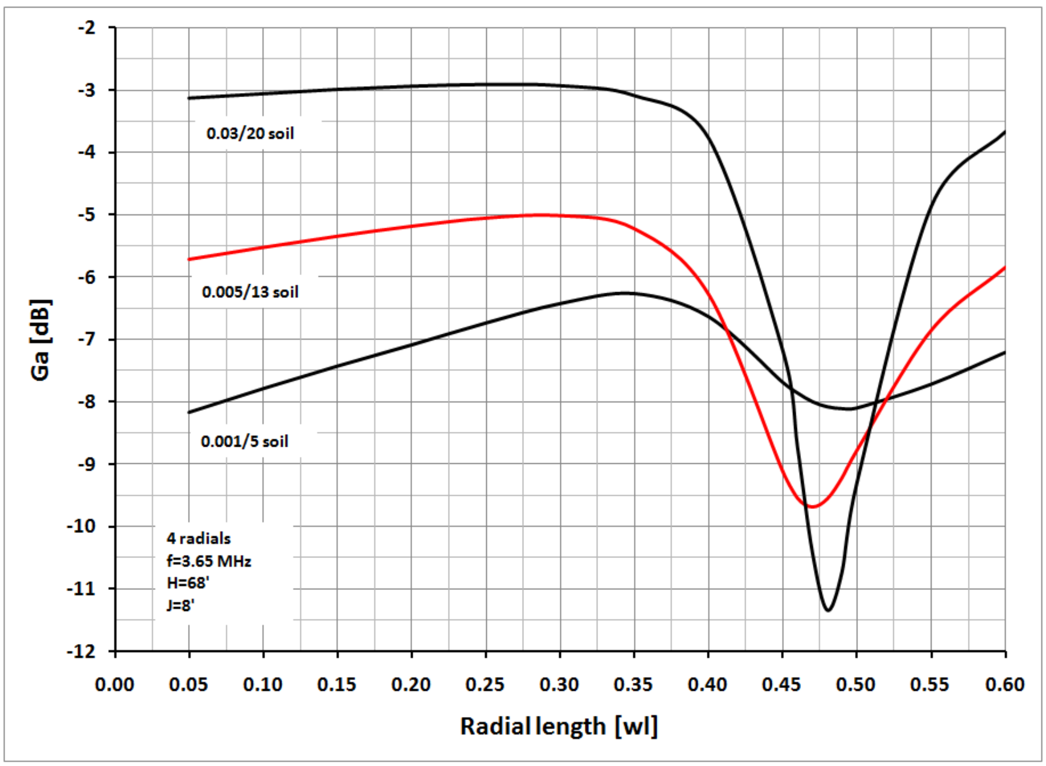

Figure 15 - Effect on Ga of different soils for H=68', J=8' and N=4.

As we saw in Figure 6, close proximity to ground has great effect on the radial resonant frequency. Belrose, VE2CV, has modeled Ga for radial spacings close to ground and the effect of different numbers of radials[42] as shown in Figure 16. Note: the data points in the graph were taken from Belrose and regraphed.

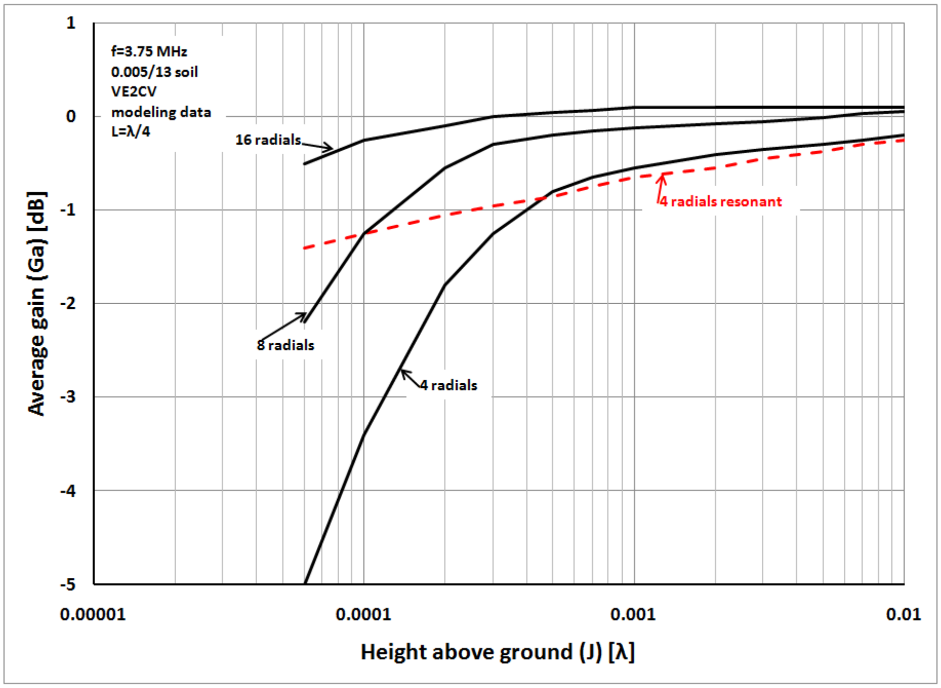

Figure 16 - Average gain when radials are placed close to ground.

The dashed line in Figure 16 represents the case where the lengths of the four radials is adjusted so that the radials are resonant. The predictions in Figure 16 agree with the experimental work shown in Figure 14 that show the effect of shortening the length of radials close to ground. Figure 16 also predicts that even a very small increase in height above ground for the radials will make a large difference in loss, especially if N is small. This large change in Ga with small elevations has been verified experimentally[3] as shown in Figure 17.

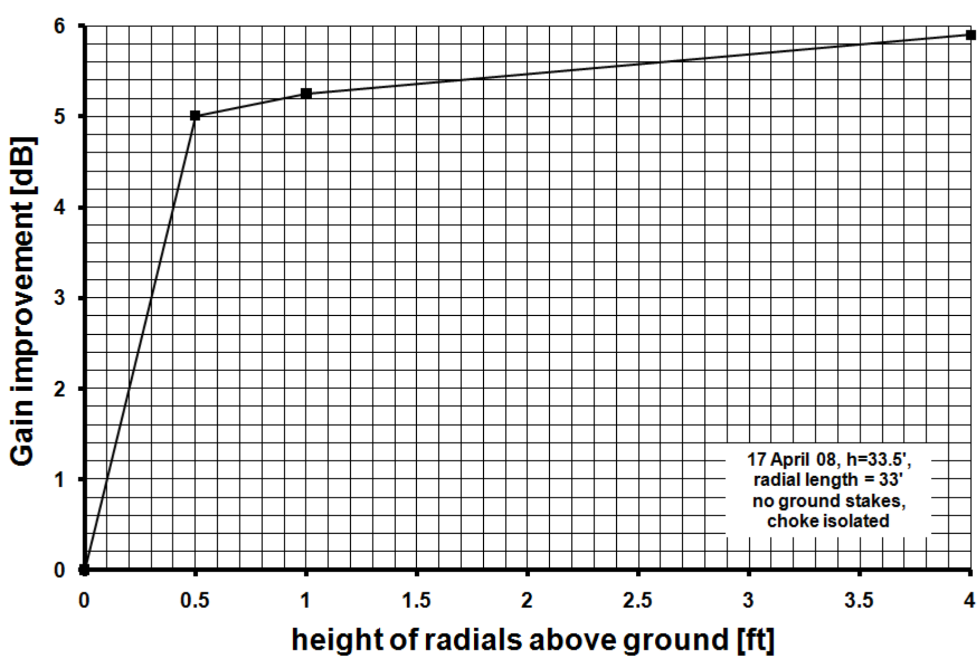

Figure 17 - Measured change in gain as four radials are elevated above ground.

In some cases it may be necessary to use a vertical with H other than λ/4. Figure 18 shows Ga as a function of L for H= 100' (≈3λ_o/8), H=68' (≈λ_o/4) and H=34' (≈λ_o/8) with and without top-loading wires. Compared to H=68', the notch for H=34' begins a lower value of L and is much deeper. Putting a short base loaded vertical over a ground-plane may not be a good idea (note: this is something that needs to be explored further). If we add two top-loading wires that restore the resonance of the 34' wire to that of the 68' wire, Ga is greatly improved. With the top-loaded vertical the peak value for Ga is a few tenths of a dB lower than for the full height vertical but that may be acceptable because the vertical is only half as high. That's something to think about for 160m verticals. It is also interesting to note that the taller vertical (H≈3λ/8) while more tolerant of longer radials is somewhat less efficient (≈-0.5 dB). The lesson to draw here is that using elevated ground systems with short verticals can be problematic but really tall verticals may not be all that great either. You have to model the specific situation carefully to make sure you understand what's going on.

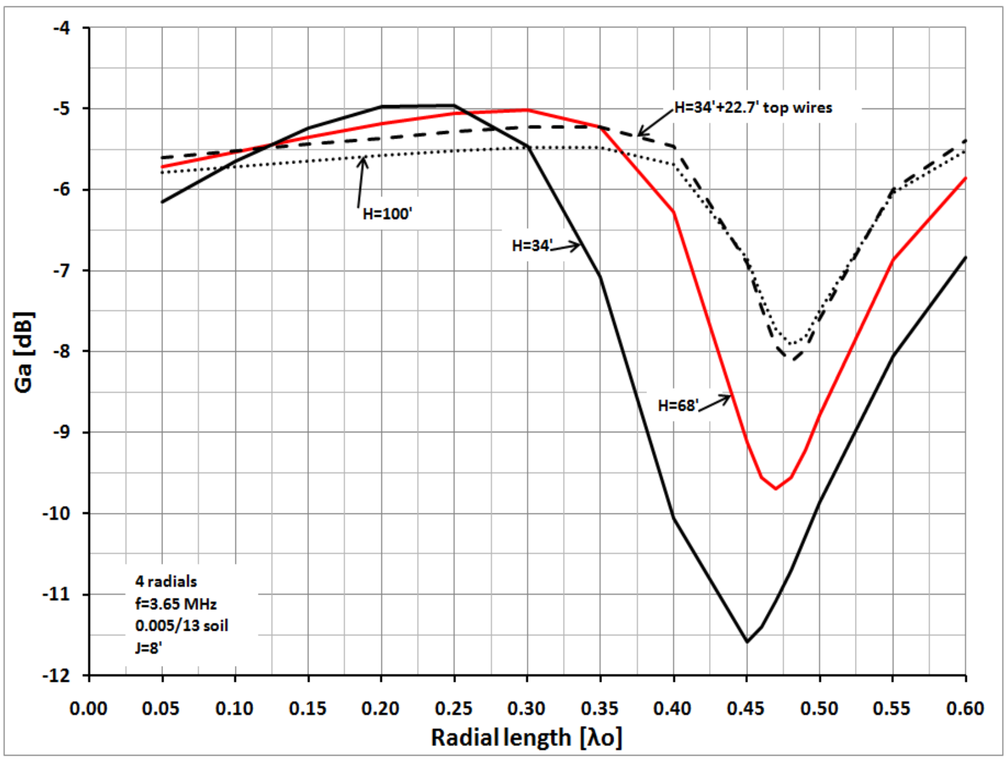

Figure 18 - Effect on Ga of short verticals. H=100', 68', 34' and 34' with top-loading.

The graphs in Figure 12 assume that H is constant. We could also have varied H so that Xin = 0 for every value of L. This may give us some insight into optimum combinations (with regard to Ga!) of H and L. Figure 19 shows what happens when we do this compared to the case where H was constant for N= 4 and 16. The curves for a fixed H (solid lines) and variable H (dashed lines) are very similar except that for the four radial case the dip sets in a bit earlier and is somewhat deeper. The maximum Ga point is about 0.28 λ_o with four radials and about 0.35 λ_o with sixteen radials. But in both cases the maximum is very broad. As long as you stay below the point where Ga starts to fall the value of L is not critical.

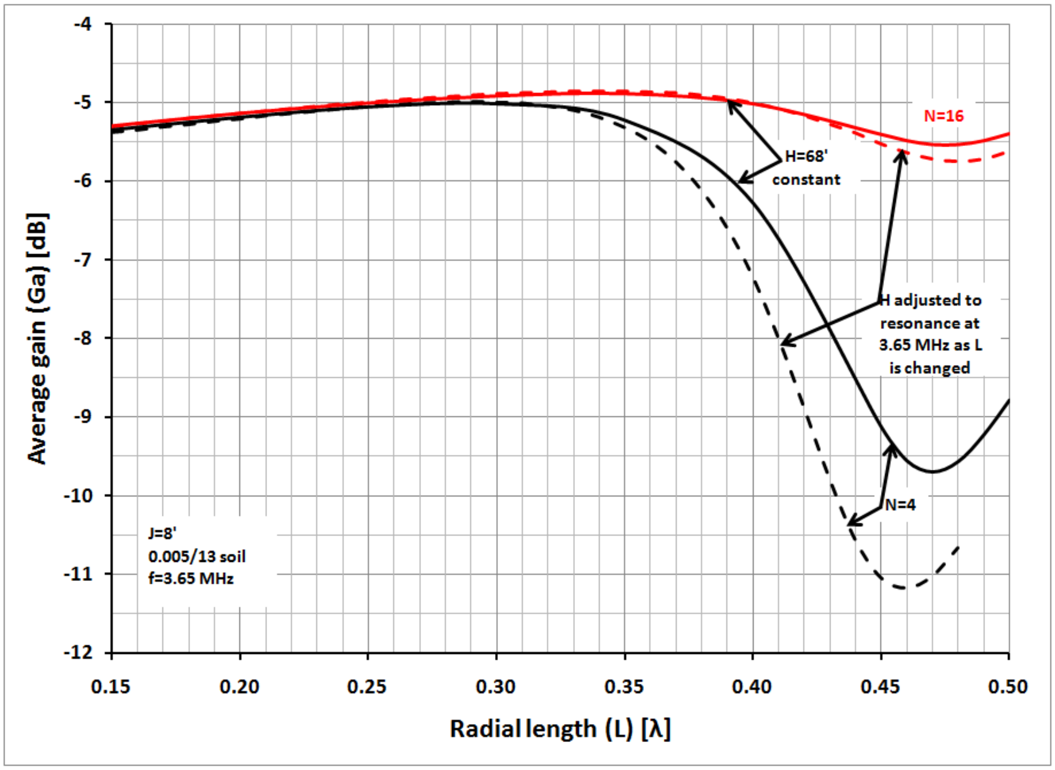

Figure 19 - Effect on Ga of radial length when H is varied to keep Xin = 0 at 3.650 MHz compared to the case where H is constant at 68' (from Figure 12).

Figure 20 shows the values for H that result in resonance at 3.650 MHz for each radial length in Figure 19.

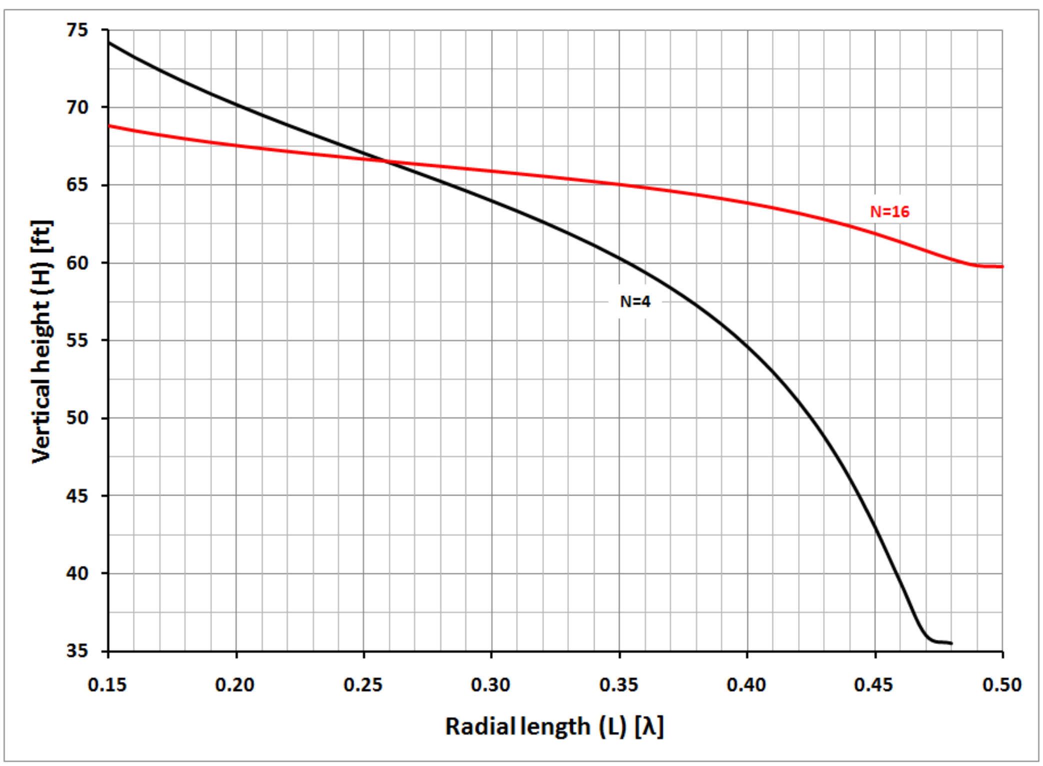

Figure 20 - Values for H that make Xin=0 as L is varied.

Again we see that the sensitivity to radial length is much smaller when more radials are used. We can also look at the effect on Rin at resonance as we vary the H+L combination. An example is given in Figure 21.

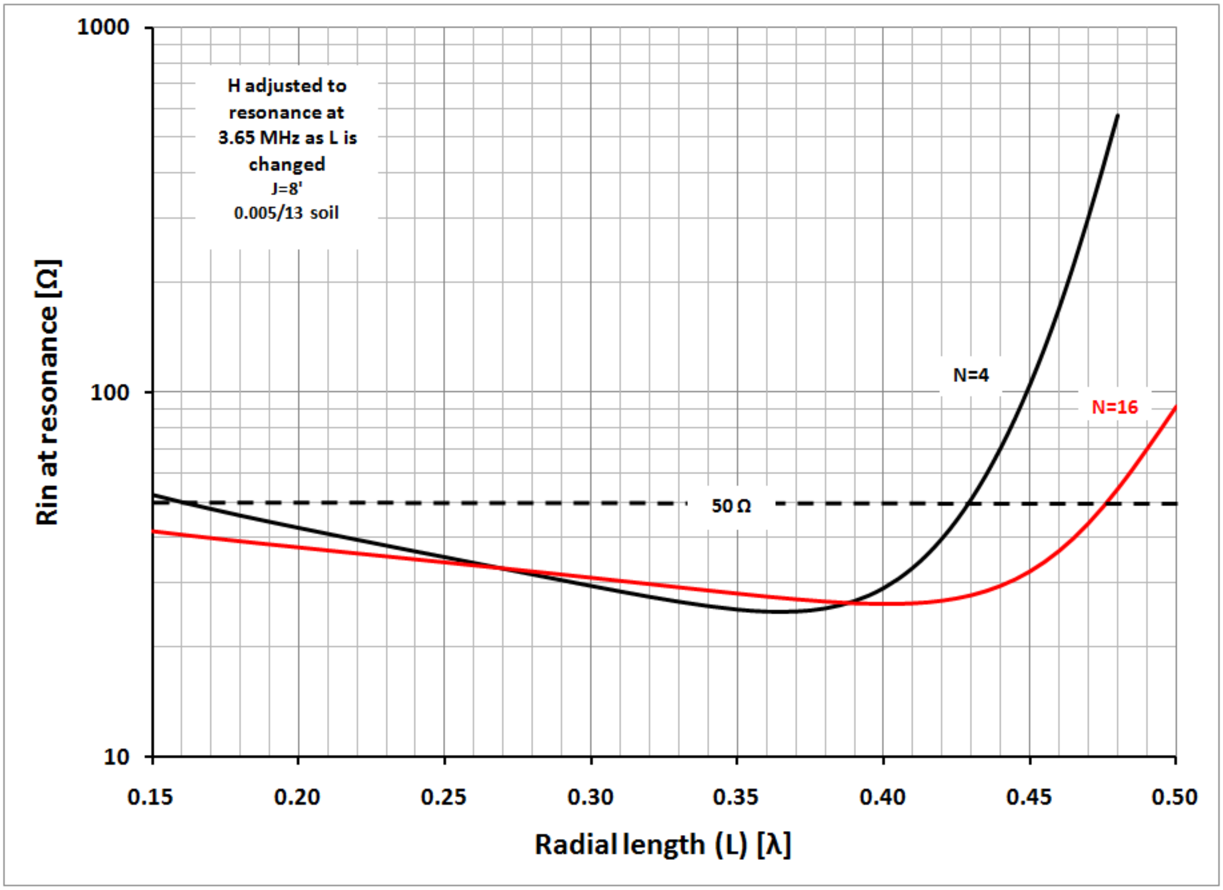

Figure 21 - Rin at resonance as a function of L.

When four radials are used there is also an important effect on the radiation pattern when the radials are too long.

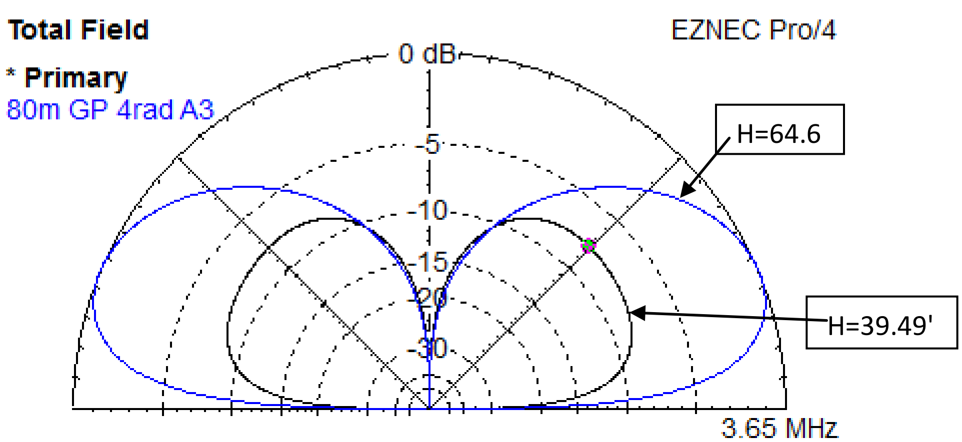

Figure 22 - Radiation pattern for H=64.64'-L=78.15' and H=39.49'-L=123.96. N=4 in both cases.

Figure 22 compares the radiation patterns for two different combinations: L=0.29 λ_o and L=0.46 λ_o. The first is close to the peak Ga value and the second is at the minimum of Ga. In the case of the long radials, not only is Ga much smaller but the peak of the radiation pattern has moved from about 22˚ to 45˚! Clearly if you are using only a few radials, long radials are bad idea.

An explanation for the dips in Ga

Why do we see these large dips in Ga for some values of L? We can investigate this by looking at the current distributions on the radials and the associated field intensities close to ground under the radials. Figure 23 shows examples of the current distribution on the radials as a function of distance from the base (feedpoint) for several different radial lengths 64', 70', 80', 100' & 121'). The graphs are for N=4 except for the plot where N=16 (the dashed line, L=121').

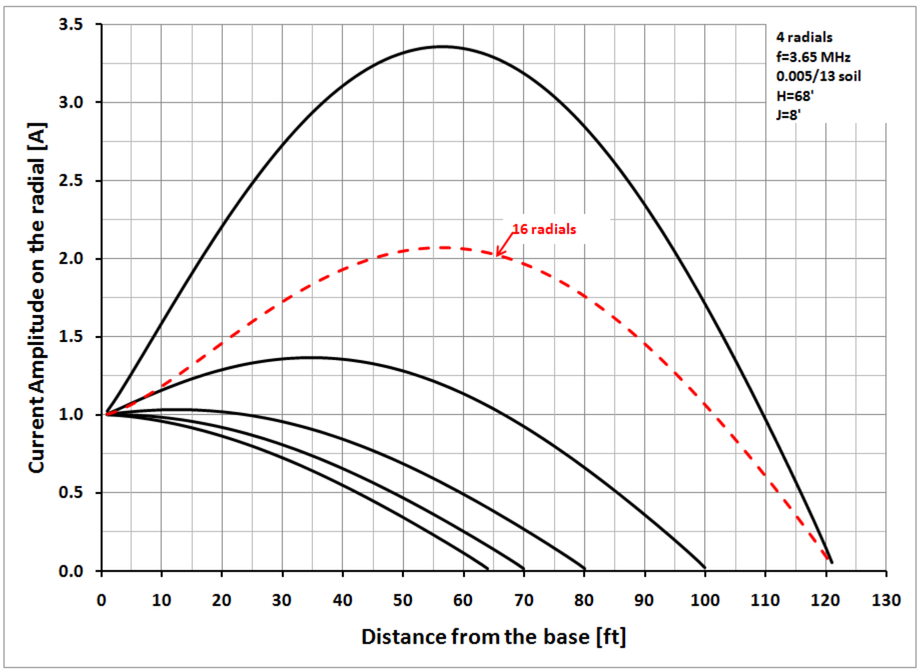

Figure 23 - Radial current distribution as a function of distance from the base. N=4, H=68', f=3.65 MHz, J=8' and average soil.

For the same current at the feedpoint, with longer radials the currents are much higher as we go out from the base. We would expect these higher currents to increase both E and H-field intensities at ground level under the radials. Using the near-field plotting capability of NEC we can visualize the field intensities as shown in Figure 24.

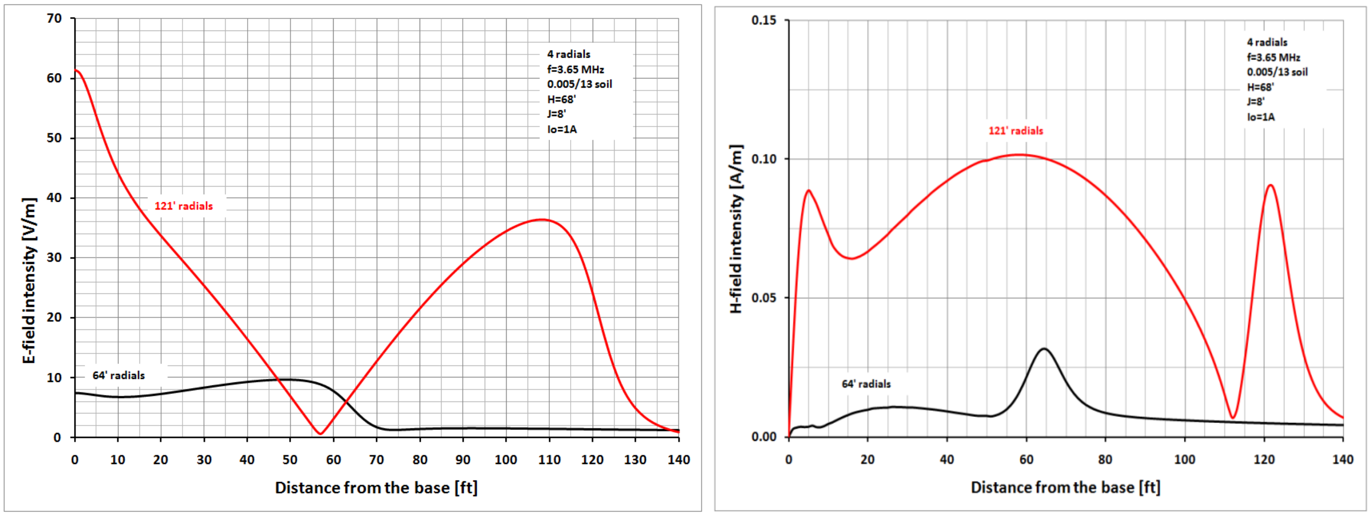

Figure 24 - E & H field intensities close to the ground surface directly below the radials with N=4.

Figure 24 shows the drastic increase in field intensities with longer radials. In this case I've chosen the longer radial length (121') to correspond to the dip in Ga in Figure 12. Since the power dissipation in the soil will vary with the square of the field intensity it's pretty clear why the efficiency takes such a large dip when the radials are too long. Figure 25 illustrates what happens to the fields under the radial fan when more radials are employed.

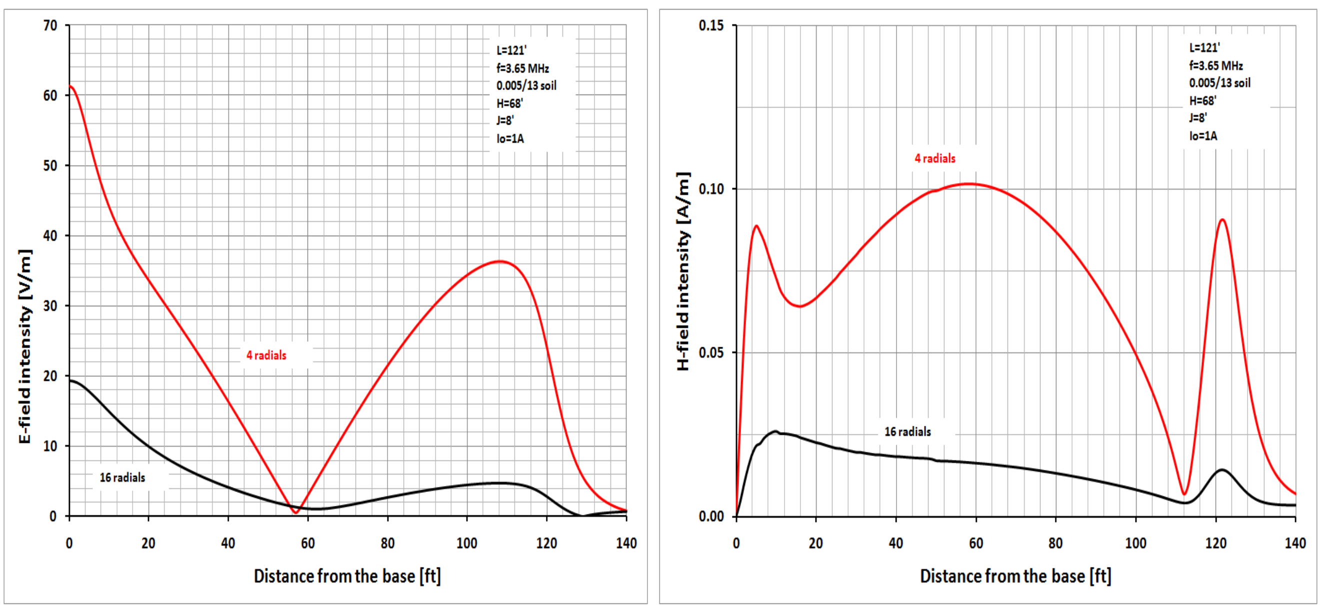

Figure 25 - E & H field intensities close to the ground surface directly below the radials. N=4 & 16.

The earlier quotation from Laport stated that the use of more radials would make the fields under the radial fan more uniform. Figure 25 certainly supports that but we can go one step further to show how much the fields are smoothed with more numerous radials. Figure 26 makes that point.

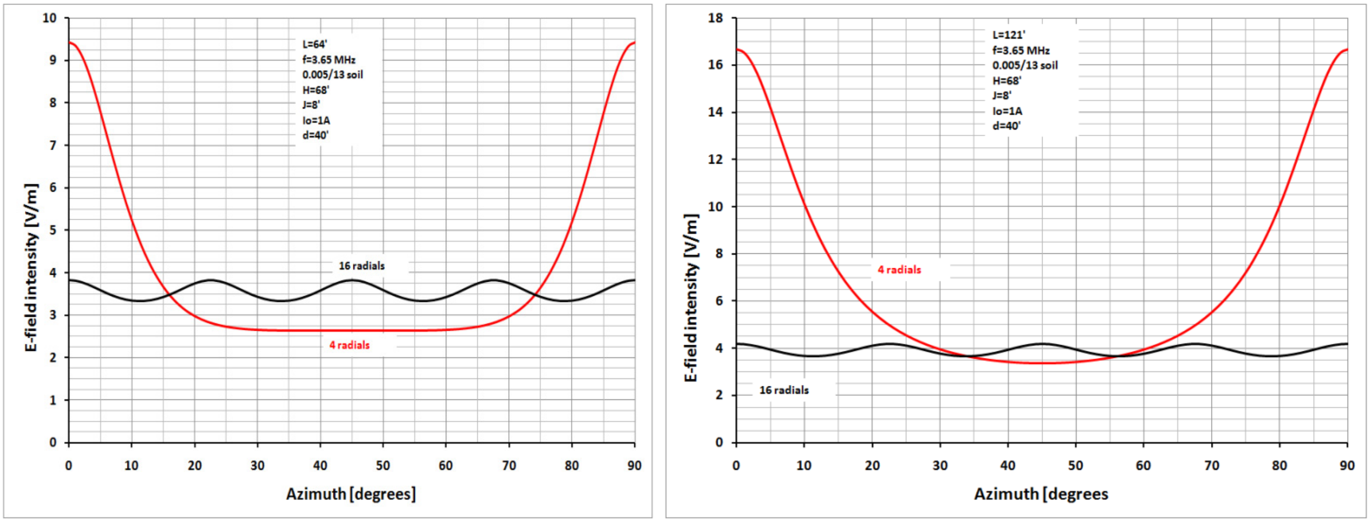

Figure 26 - E-field intensity just above ground on a 90˚ arc 40' from the base.

Figure 26 is the E-field intensity just above ground level at points lying on an arc with a radius of 40' (centered on the base) for two radial lengths (L=64' and 121'). We can see that with only four radials the E-field peaks sharply directly under the radials but with 16 radials the field is much more uniform.

Multiband verticals

For a single band antenna we can avoid the problems of long radials by simply using radials that are short enough or by increasing the number of radials but what about the case of multi-band antennas where you typically have four λ/4 radials for each band? For example, if you have 40m λ/4 radials these will be λ/2 on 20m, 3λ/4 on 15m, etc. In light of the information we've just seen for Ga as a function of L, is that a problem? I don't have the space here to explore that in detail with modeling but I have looked at multi-band elevated verticals experimentally. The information is in reference [5] that is available on my web page (www.antennasbyn6lf.com). The experimental work indicated that as long as there are a large number of radials (whether they are the same length or of different lengths) then you don't have a problem but if you try to use only a few long radials you will have problems. Read the article for the details. This is yet another example where not stinting on the radials is a very good idea.

Potentials on the radials

Elevated radial systems can have very high voltages between the radial wires and ground. Figure 27 shows examples of the voltage from a radial wire to ground for ideal 4, 12 and 32 $\lambda_o/4$ radial systems.

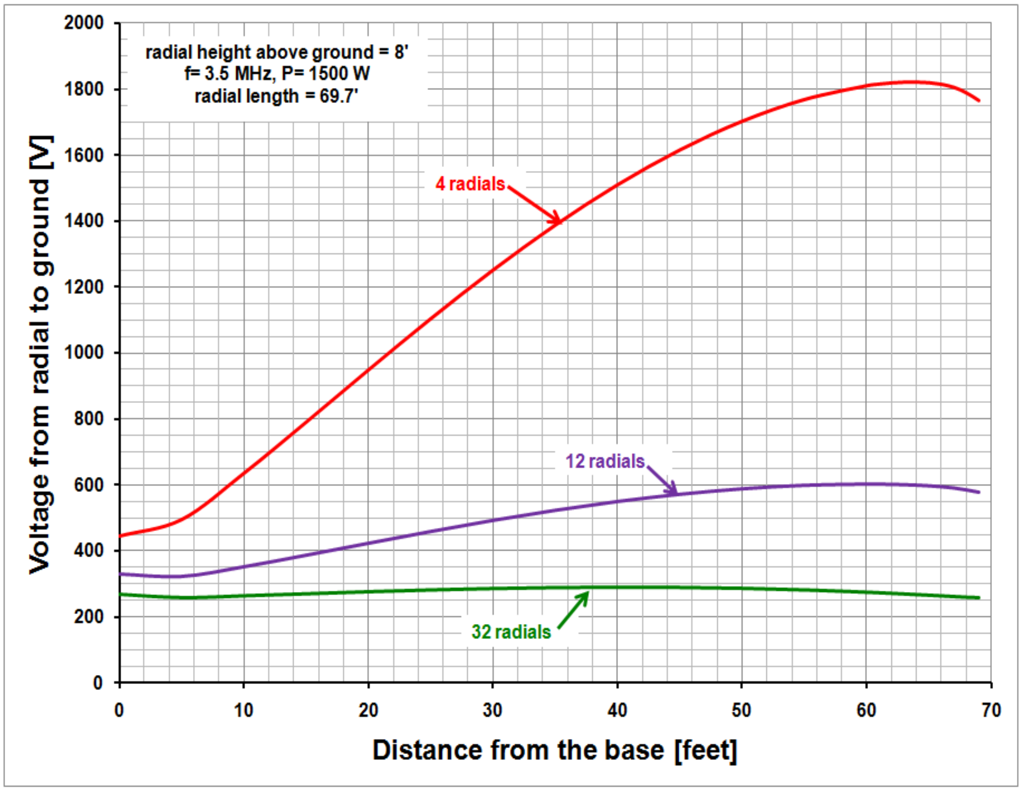

Figure 27 - Examples of the voltage from a radial wire to ground with different numbers of radials. The input power to the vertical is 1500 W, the operating frequency is 3.5 MHz and the radial system is elevated 8' above ground.

I think this Figure makes it clear why you want to keep the radials out of reach! Note that as more radials are added the potential difference between the radials and ground drops significantly and becomes more uniform as we go away from the base of the antenna. This is a reflection of the reduction in E-field amplitude with more numerous radials as shown in Figures 24-26. But even with a large number of radials that voltage is still high. This voltage will vary with the square root of the power level so that going down from 1500 W to 100 W, a change of 15:1 (0.067), the voltage only drops by 0.26! Be careful!

Feedpoint impedances

The behavior of the feedpoint impedance over the band (3.5-3.8 MHz for these examples) as we vary H, L, J, N and soil characteristics is an important factor. The point I want to make in this section is how widely the input impedance of ground-plane antenna can vary as we change one or more of the variables. There is no one number for Zin! We will also look at variations in SWR bandwidth.

A graph of the feedpoint impedance (Zin = Rin +j Xin) from 3.5 to 3.8 MHz for different numbers of radials is shown in Figure 28. Note that in Figures 28-31 that H = L and is adjusted so that the model is resonant at 3.65 MHz for each variation of parameters. As the parameters N, J and soil characteristics are changed the values for H and L vary somewhat. From Figure 28 it can be seen that N has a very strong effect on the feedpoint impedance (Zin) although that effect diminishes as N increases. As shown in Figure 29, we can convert the information in Figure 28 to SWR. In this case the Zo impedance for the SWR calculation is taken to be Rin at resonance (3.65 MHz) for each value of N.

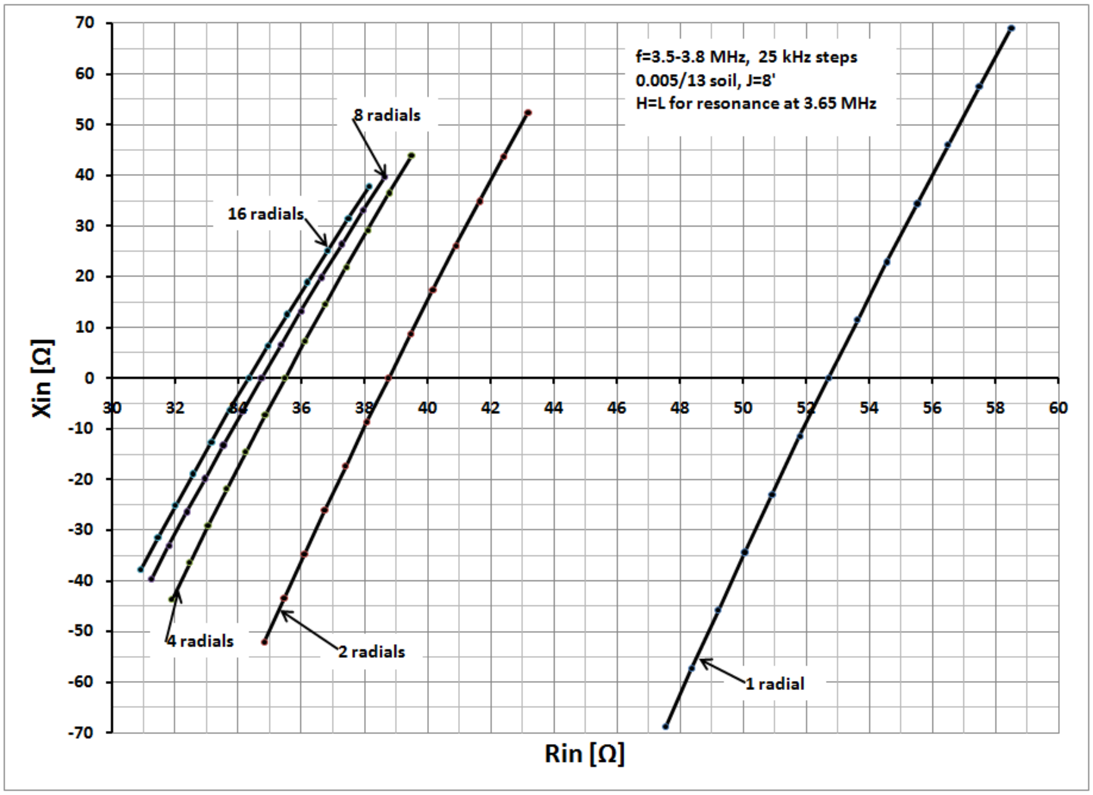

Figure 28 - Xin versus Rin (Zin = Rin + j Xin) where frequency varies from 3.5 MHz (lower left ends of the curves) to 3.8 MHz (upper right ends of the curves) for different values of N (radial number). Frequency is stepped in 25 kHz intervals.

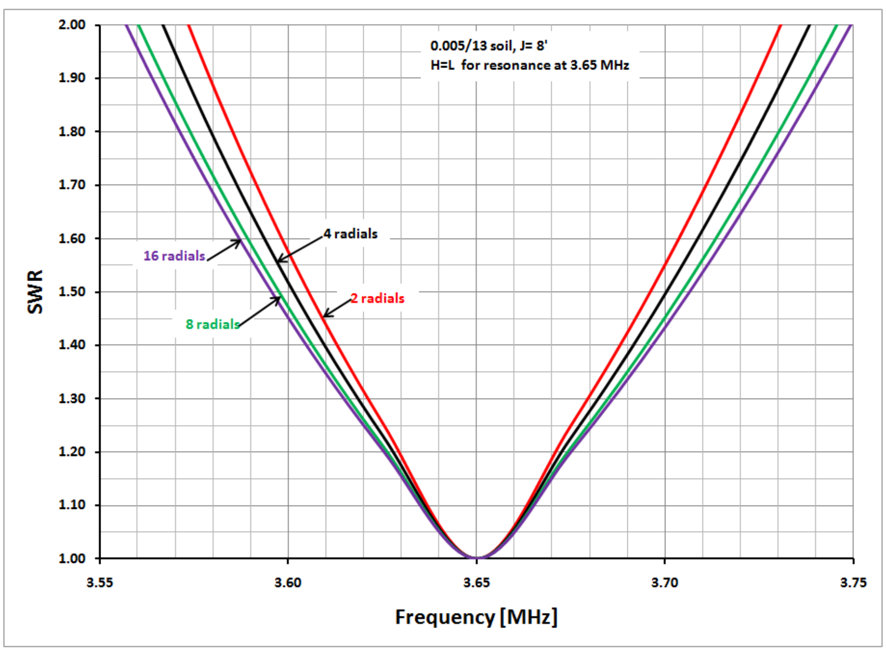

Figure 29 - Feedpoint SWR as a function of N.

Figure 29 shows that the 2:1 SWR bandwidth increases somewhat as N is increased but N=16 approaches the point of vanishing returns for bandwidth.

Figure 30 shows the effect of height above ground of the radial fan (J) on Zin for N=4.

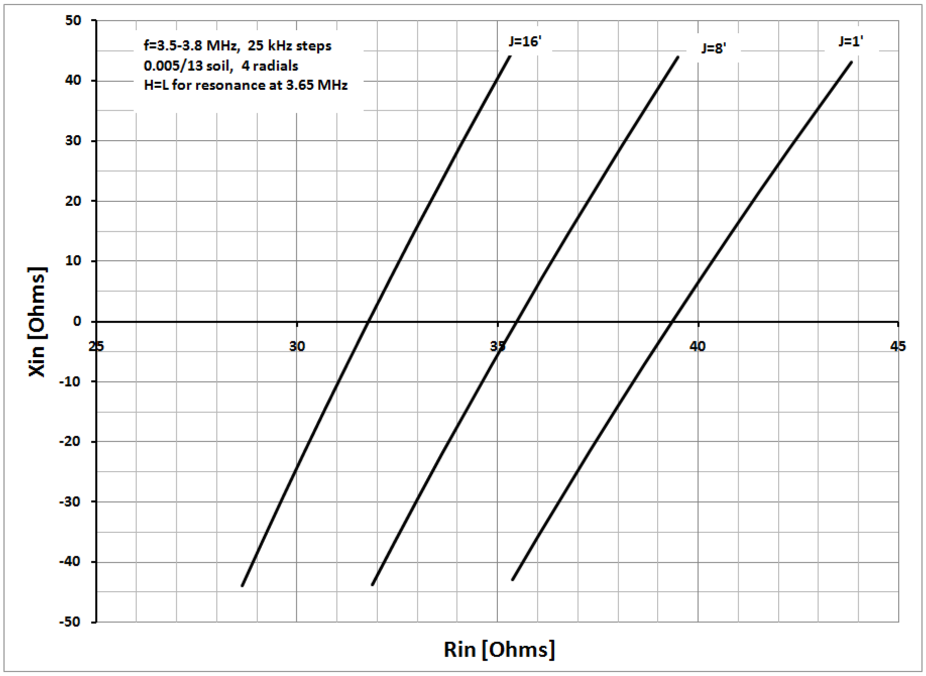

Figure 30 - Effect of height above ground.

It's pretty clear that the value for J has a strong effect on Zin. The effect of different soil characteristics for a given value of J (8' in this example) is shown in Figure 31.

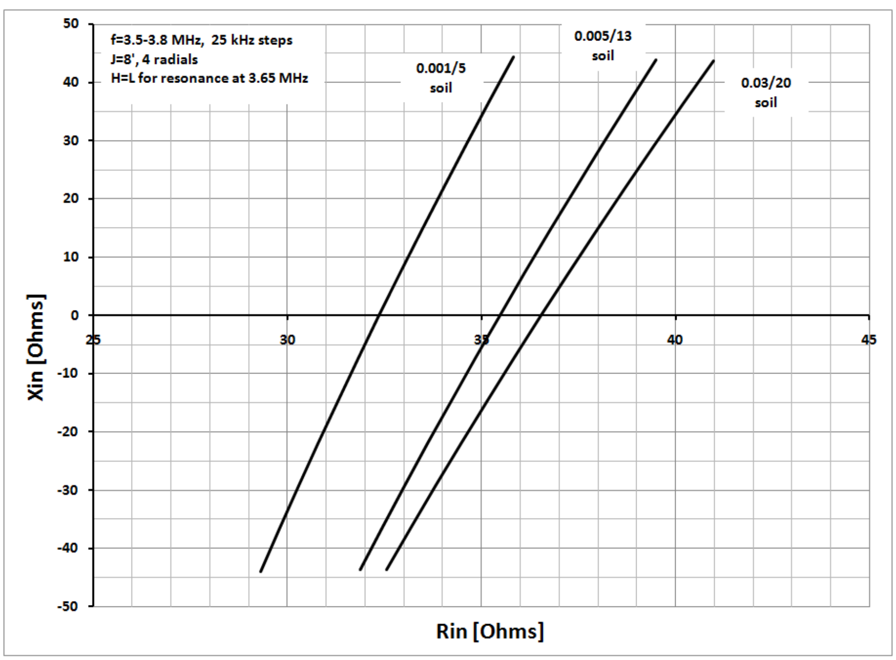

Figure 31 - The effect of different soil characteristics on Zin.

The information in Figures 28-31 represent only a few possible combinations but they make the point that the feedpoint impedance of an elevated radial vertical is a strong function of all the variables so that each installation is unique.

We can also see the behavior of Zin over the band for different combinations of H and L that are resonant at 3.65 MHz. Some examples of Zin are given in Figure 32 and the associated graphs for SWR, are given in Figures 33 and 34. N=4 and the H&L combinations are shown on the graphs.

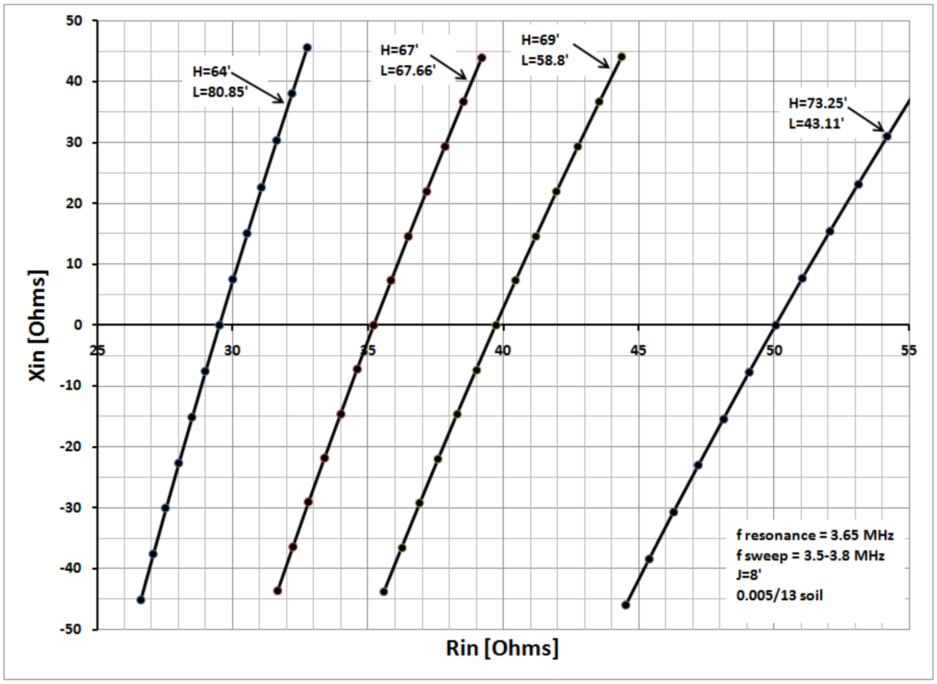

Figure 32 - Zin variation for different combinations of H and L that are resonant at 3.65 MHz.

The combination H=73.25' & L=43.11' has the very nice property that Zin = 50 Ω at 3.65 MHz. As shown in Figure 33, this results in a relatively wide 2:1 SWR bandwidth compared to the other combinations.

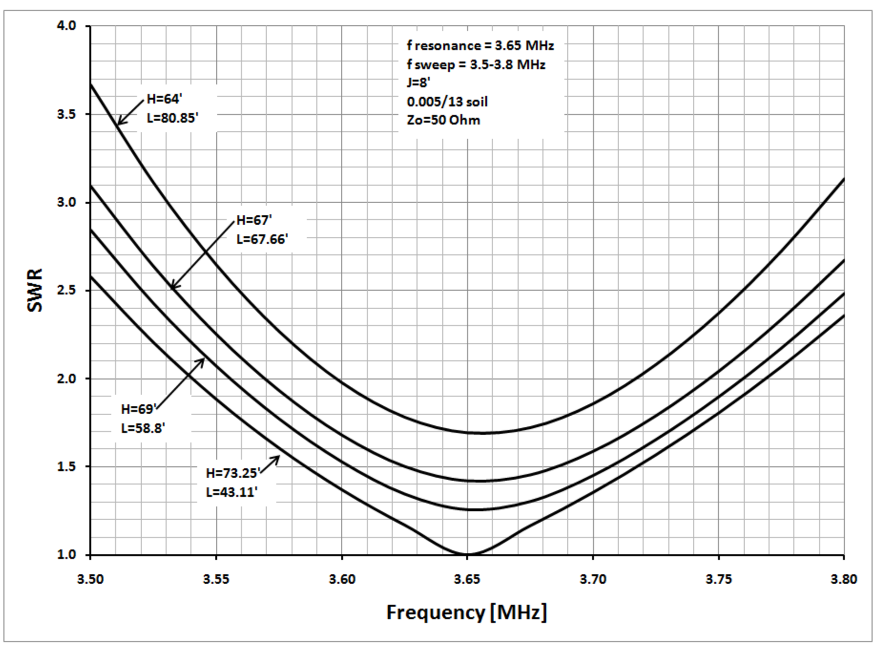

Figure 33 - SWR for various combinations of resonant H and L. Zo = 50 Ω.

The greater match bandwidth is not just because Zin = 50 Ω at resonance. The combination also has intrinsically more bandwidth as shown in Figure 33 where the Zo at resonance is set to Rin at resonance for each combination of H and L separately.

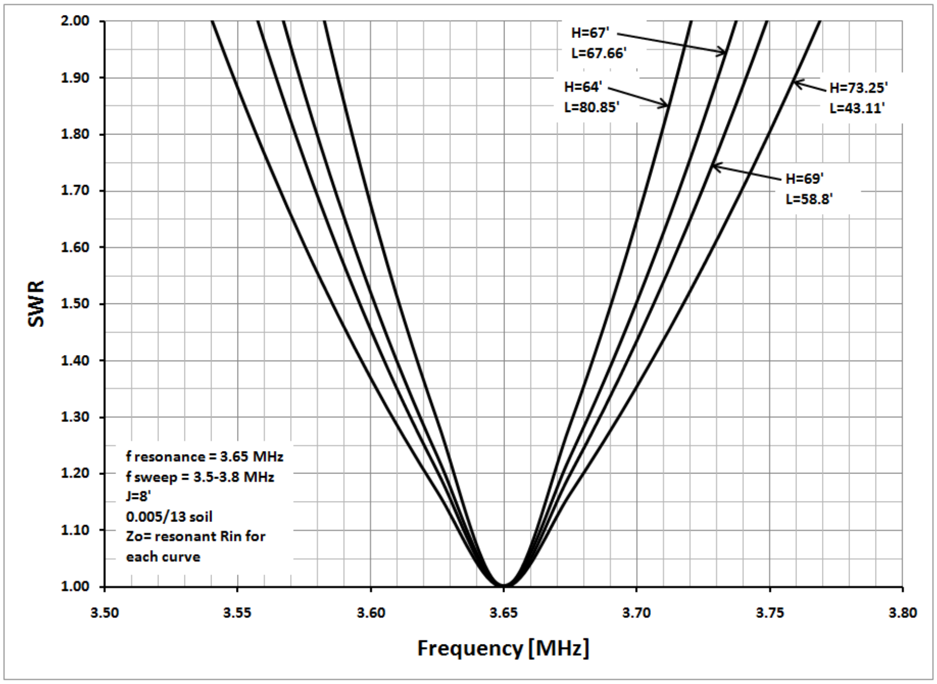

Figure 34 - SWR with Zo equal to Rin at resonance for the particular combination of H and L.

The idea increasing the feedpoint impedance at resonance to 50 Ω by making the vertical taller and the radial fan radius smaller has actually been around for many years: Rin at resonance can be increased by sloping the radials downwards from the base. In effect you are making the vertical taller and reducing the radial fan radius which is what we did in the above example.

Figure 9 showed how Lr varied for different values of N and H. For H=69', Lr decreased rapidly as more radials were added. We can play this game to find designs where Zin = 50 Ω at resonance. Figure 35 is a graph where L is varied from 15' to 100' for two values of H (72' & 77.6"). Note that H in the range of 72'<=>77.6' represents the limit which allows Rin = 50 Ω. Longer or shorter values for H do not have a point where Rin = 50Ω for L =15'<=>100'. The combination of H=72', L=25', N=16, J=8' over average ground will give us Zin = 50 Ω at 3.65 MHz. Figure 36 shows the comparison for SWR between two combinations where N= 4 and N=16. This illustrates one of the advantages of using more radials.

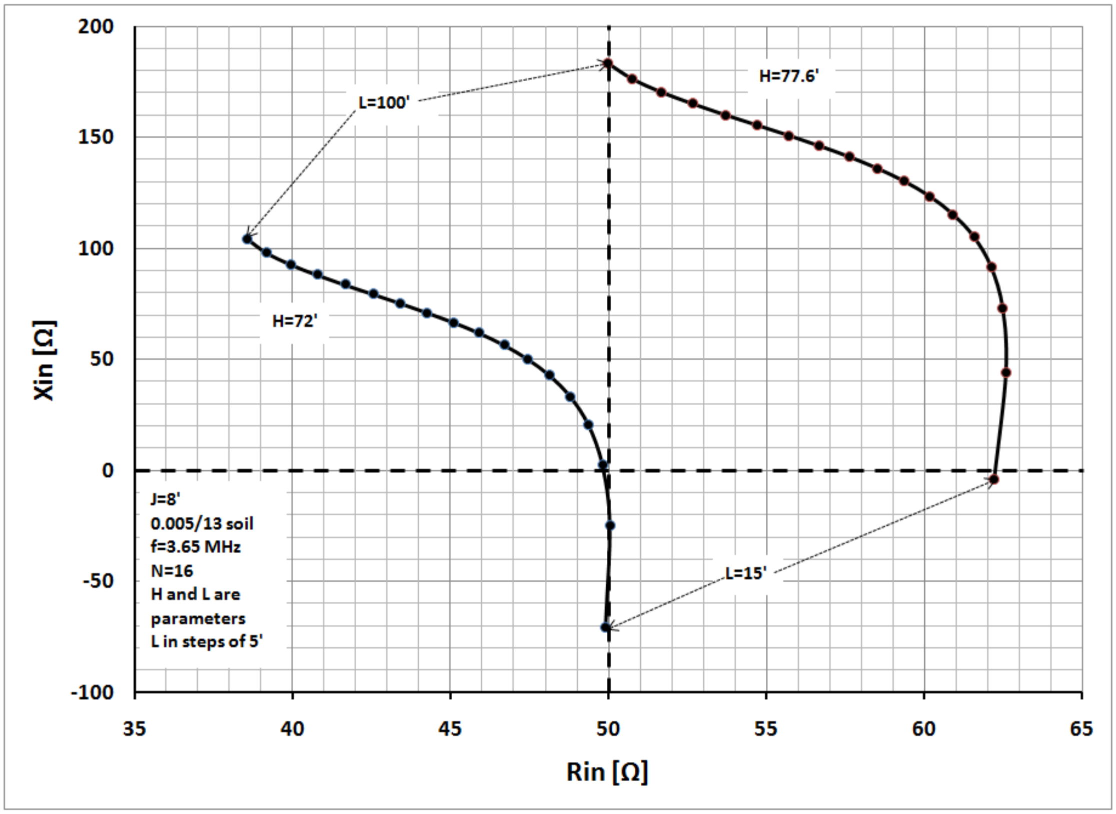

Figure 35 - Zin as a function of radial length (L) for H=72' and 77.6'. N=16.

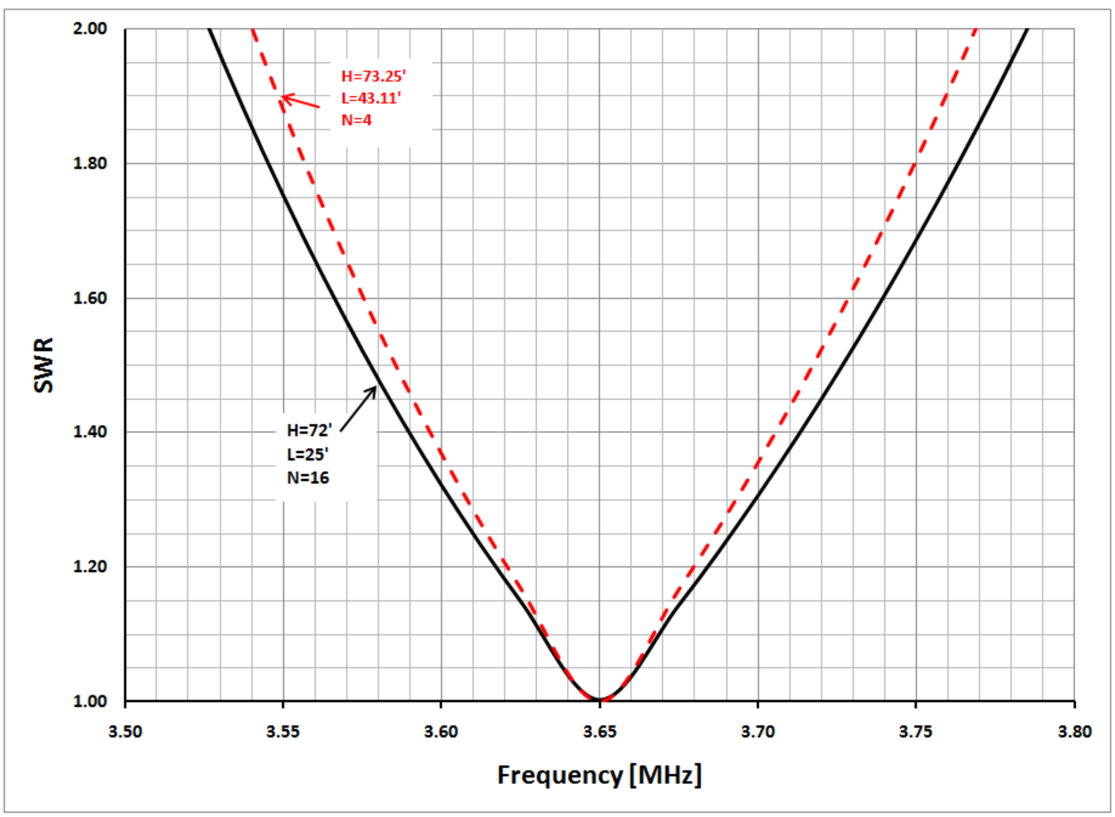

Figure 36 - SWR over 3.5-3.8 MHz for two different combinations of H and L.

For H= 72' and N=16, L is only 25' that represents a drastic reduction in the radius of the radial fan. In exchange for an increase in height on the order of 6' we have a good match over a wide portion of the band and a small diameter radial fan. Instead of increasing the height we could have just added a couple of short sloping top-loading wires. This is all very nice but it's not entirely for free. When compared to the normal four radial system (H=67', L=67.7') Ga for the H=72', L=25' combination is lower by about 0.25 dB. You sacrifice a small amount of gain. Whether that is acceptable for the improvement in matching is an individual decision.

Weber has made a suggestion that overcomes the reduction in gain associated with small radial length[43]: use longer radials, this will result in Xin≠0 but you can tune out the reactance with a series impedance. He has also pointed out that if Xin is inductive (+) then you can tune out the reactance with a series capacitor at the feed point. Looking back at figure 35 we see that this trick will work for H>72' (that's for this particular case where N=16, J=8' over average ground!). If we chose H=75', L=70', N=15 and adjust the series capacitor at the feedpoint as we move across the band, we get the result shown in table 1. Note that Xin is given in the table but Cs tunes it out.

Table 1, Zin and SWR from 3.5 to 4.0 MHz for H=75', L=70' and n=16.

Frequency [MHz]	Rin [Ω]	Xin [Ω]	Cs [pF]	SWR
3.50	43.7	69.6	654	1.14
3.65	49.4	113.7	384	1.01
3.80	56.0	159.4	263	1.12
4.0	66.6	223.6	178	1.33

What we see here is a vertical which can have a very low SWR across the entire 75/80 m band. It isn't necessary that Cs be adjusted at every point. Three or four values of Cs switched with relays would probably still provide acceptable SWR over the entire band. For the case where H=72', L=25' and N=16, Ga=-5.52 dB. When we change to H=75', L=70' and N=16, Ga=-5.03 dB. That's an improvement of +0.5 dB in signal strength!

There is another option to make Zin = 50 +j0 Ω at resonance. Instead of making the antenna taller (or top-loading it) and the radials shorter, you can simply shift the feedpoint up into the vertical to a point where Rin = 50 Ω. This just a matter of moving the base insulator up into the antenna. You won't get quite as much match bandwidth as with the taller vertical but it will be close and you can use longer radials that give a better Ga.

All the examples to this point have assumed that the excitation at the base of the vertical was isolated from ground: i.e. a choke (balun) was used in series with the feedline. If a choke is not used and the coaxial feedline is simply connected to the antenna and run down to ground, usually with the shield to the radials and the center to the vertical, there will be additional ground currents which increase loss. In a 4-radial elevated system Ga typically falls -0.25 to -0.5 dB or even more for lossy soils if a choke is not used. If 12-16 radials are used the increased loss is much smaller, usually only a few tenths of a dB. You might argue that when N is large a choke is not needed but I think it wise to be cautious and use a choke even in that case.

Earlier we saw how the radial length (L) affected the efficiency (Ga) of the antenna. We also saw that the effect was reduced when more radials were used. It is useful to look at Zin as both N and L are varied, especially around values of L near $\lambda_o/4$. Figure 37 shows the effect of varying L on Xin.

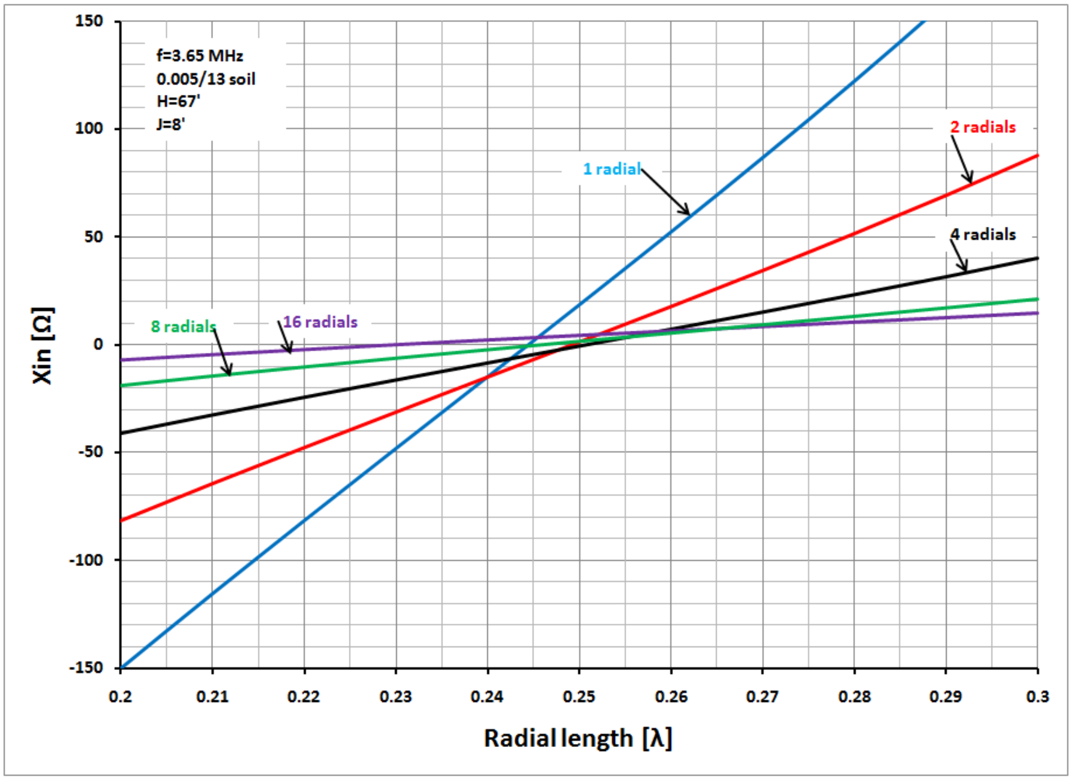

Figure 37 - Effect of changing L in the neighborhood of λ/4 as a function of radial number.

Figure 37 is particularly interesting in that is shows how sensitive the Xin component of Zin is to radial length when only a few radials are used. The Rin component is not nearly as sensitive. This becomes important when we look at current asymmetries in the radials. Adding more radials reduces the sensitivity of Zin to radial length and also the susceptibility to radial current asymmetry. Weber[43] generated a graph very similar to figure 37 by assuming the radials were open circuit transmission lines and plotting the impedance at the feedpoint as more radials were added in parallel. More on radials as transmission lines in the next section.

EFFECT OF ASYMMETRIES IN THE RADIAL FAN

Is there significant current division asymmetry among the radials of typical installations and, if there is, do we need to be concerned about it? To answer the first part of this question, Dick Weber, K5IU, made a series of measurements on representative 80m and 160m verticals with two and four elevated radials[10]. I have summarized some of his data in table 2.

Table 2, radial current comparisons from K5IU measurements[10].

			Relative current	Relative current	Relative current	Relative current
Antenna #	Station ID	Frequency [MHz]	Radial 1	Radial 2	Radial 3	Radial 4
1	K5IU	3.528	1.00	0.52	0.27	0.27
1	K5IU	3.816	0.96	1.00	0.51	0.51
1	WX0B	1.805	1.00	0.01	-----	-----
1	WX0B	1.885	1.00	0.05	-----	-----
2	WX0B	1.805	1.00	0.80	-----	-----
2	WX0B	1.885	1.00	0.10	-----	-----
1 modified	WX0B	1.805	1.00	0.83	-----	-----
1 modified	WX0B	1.885	1.00	0.76	-----	-----
1	W7XU	1.805	1.00	0.00	0.00	0.00
1 modified	W7XU	1.805	0.03	1.00	0.10	0.07
final	W7XU	1.805	1.00	0.00	0.00	0.00
final	W7XU	1.900	0.03	1	0.10	0.07

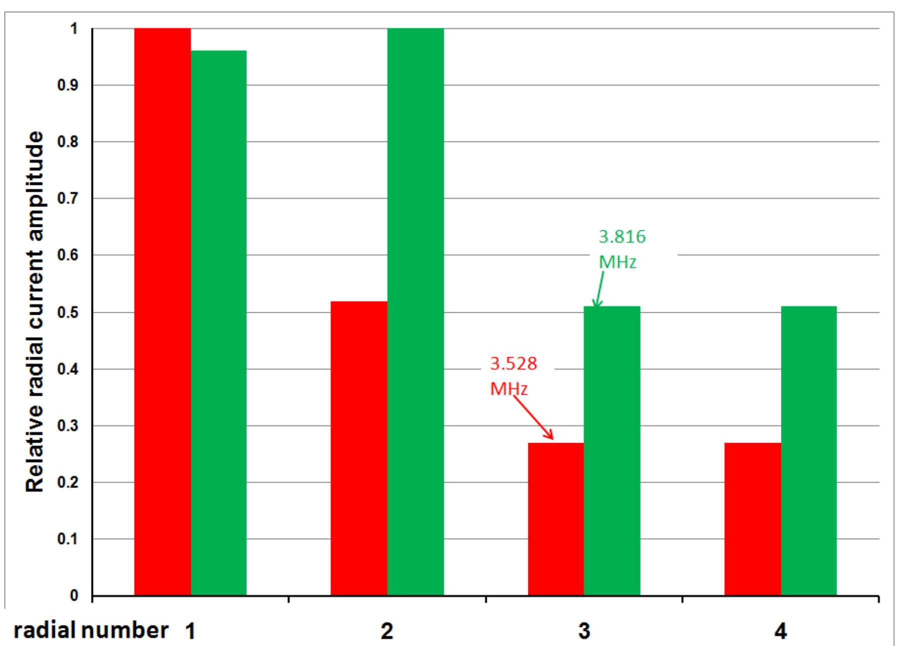

Figure 38 - Current division between the four radials at 3.528 and 3.816 MHz for the 80m vertical at K5IU.

Data tables are helpful but sometimes a graph of the data has more impact. Figure 38 compares the radial current division for K5IU's 80m vertical with four radials. Figure 38 shows two things: the radial current division between the radials is far from equal and the division ratios change as we move across the band. Unfortunately this is typical of elevated ground systems with only a few radials as shown in table 2.

Weber explains this behavior by pointing out that a horizontal radial above ground is actually a section of single wire transmission line open-circuited at the far end so that in the region where $L \approx \lambda_o/4$ it acts like a series resonant circuit. Figure 39 shows an equivalent circuit.

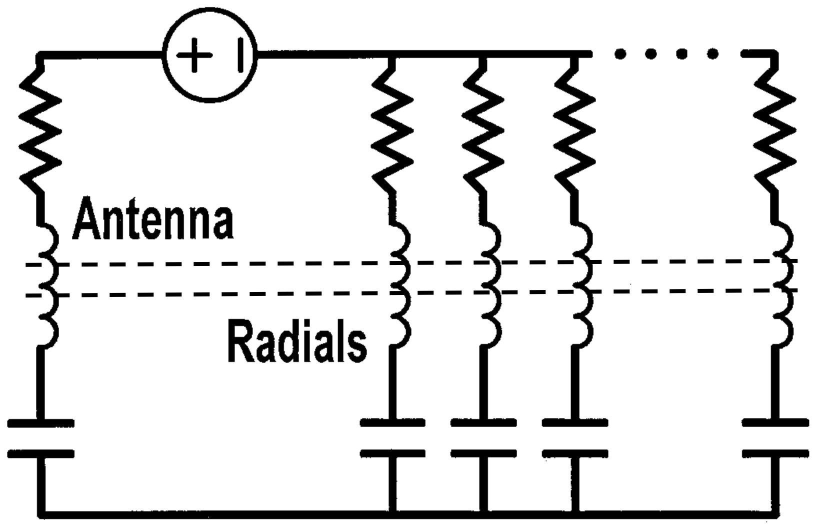

Figure 39 - Equivalent circuit for a vertical with elevated radials.

Individually the radials may have differing resonant frequencies due to length variations, varying ground characteristics under a particular radial[9], nearby conductors, etc. At a given frequency a particular radial may be close to series resonance that means it has a low input impedance and may therefore take the majority of the current. This is a reasonable idea but the basic model in Figure 38 doesn't take into account the coupling between the individual radials or between the radials and the vertical. It would be more correct to add mutual coupling between all the inductive elements of Figure 38 as shown by the dashed lines. In the case of four radials, the radials are at right angles to each other and to the vertical so that the mutual coupling is small (but not zero). When you go to eight radials for example, the angle between the radials goes from 90˚ to 45˚. That greatly increases coupling between the radials.

All this is very interesting but so what? Does current division asymmetry in the radials cause any problems we should worry about? One way to look into this is to model a system with only one radial, that might be a worst case. Several of the examples in table 2 show almost all the radial current to be in one radial. Figure 40 shows a comparison in the azimuth radiation patterns between one and four radials with J=8' and f=7.2 MHz, at an elevation angle of 22˚. Note, I have changed from 80m to 40m for the following examples simply because this work was already on hand. With four radials the pattern is symmetric within .1 dB but with only one radial the pattern is distorted with a F/B ratio of 4.6 dB. In addition the average gain for one radial is about 0.5 dB lower than Ga with four radials. There is a substantial signal reduction (almost 5 dB!) in the direction away from the single radial. Over poor soil Ga is even lower and the F/B can be 6 dB or more.

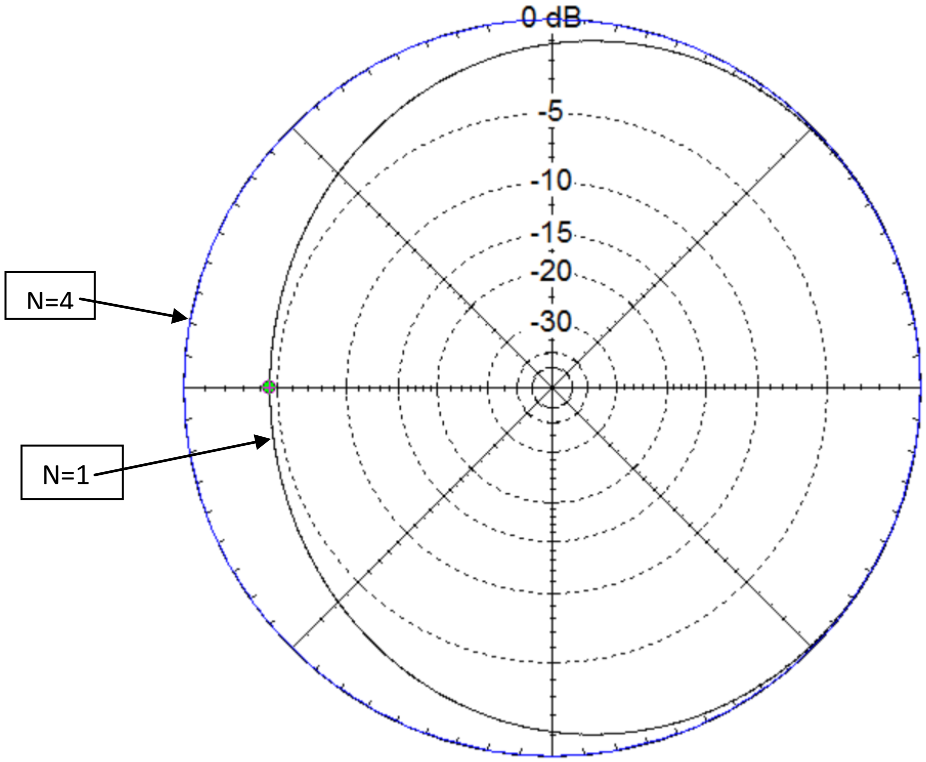

Figure 40 - Azimuth radiation pattern comparison between one and four elevated radials. J=8', f=7.2 MHz over average ground. The elevation angle for these plots is 22˚.

Does having all the current in one radial represent the worst case or can we have even more pattern distortion and/or lower Ga in some other cases? NEC modeling can be used to investigate this question. We'll start with a 40m λ/4 vertical with four radials (like that shown in Figure 4). Radials 1 & 2 form an opposing pair with a length = L. Radials 3 & 4 are a second opposing pair with length = M. First we'll model the antenna with all the radials the same length (L=M) and then with radials that differ in length (L≠M).

The feedpoint impedances (from 7.0 to 7.3 MHz) for three different radial length configurations are compared in Figure 41 as the frequency is varied from 7.0 to 7.3 MHz. The plot on the left is for the case where all the radials are identical (L=M=34.1'). The looping plot on the right is for the case where L=35.6' and M=33.1', this represents a length error of +/- 2.9%. The middle plot is for L=34.6' and M=33.6', that is a length error of +/- 1.4%. Clearly even modest radial length asymmetry can have a dramatic effect on the feedpoint impedance and resonant frequency. The resonant frequency is the point at that X_{in}=0.

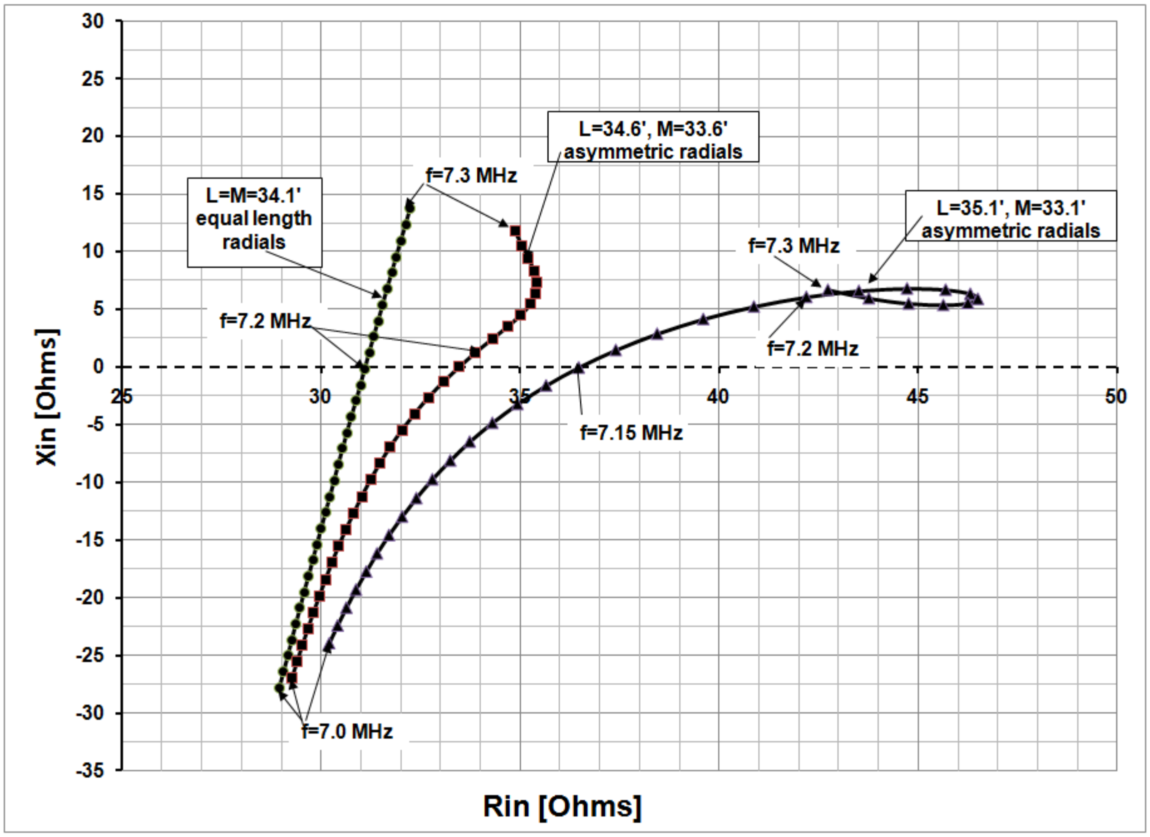

Figure 41 - A comparison of the input impedances ($Z_{in}=R_{in}+jX_{in}$) from 7.0 to 7.3 MHz at the feedpoint of the vertical for symmetric and asymmetric radial lengths.

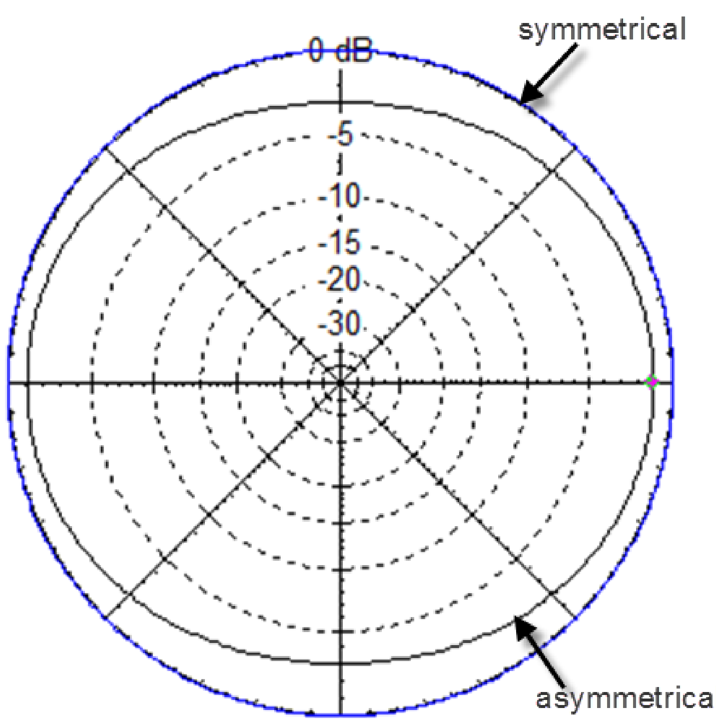

Figure 42 - Radiation pattern comparison between symmetric (L=M=34.1') and asymmetric (L=35.1', M=33.1') radials at 7.25 MHz.

Feedpoint impedance is not the only problem associated with asymmetric radial lengths. Figure 42 compares radiation patterns between symmetric and asymmetric systems at 7.25 MHz. The amount of pattern distortion varies across the band from a fraction of a dB at 7.0 MHz to 3 dB at 7.25 MHz. Besides the distortion, the gain in all directions is smaller for the asymmetric case. Computing the average gains for the symmetric and asymmetric cases, there is about a 1.6 dB difference. What this tells us is that asymmetric radials can lead to significantly higher ground losses!

Pattern distortion and increased ground loss with asymmetric radials occurs because the radial currents with asymmetric radial lengths are very different from the symmetric case. An example is given in Figure 43.

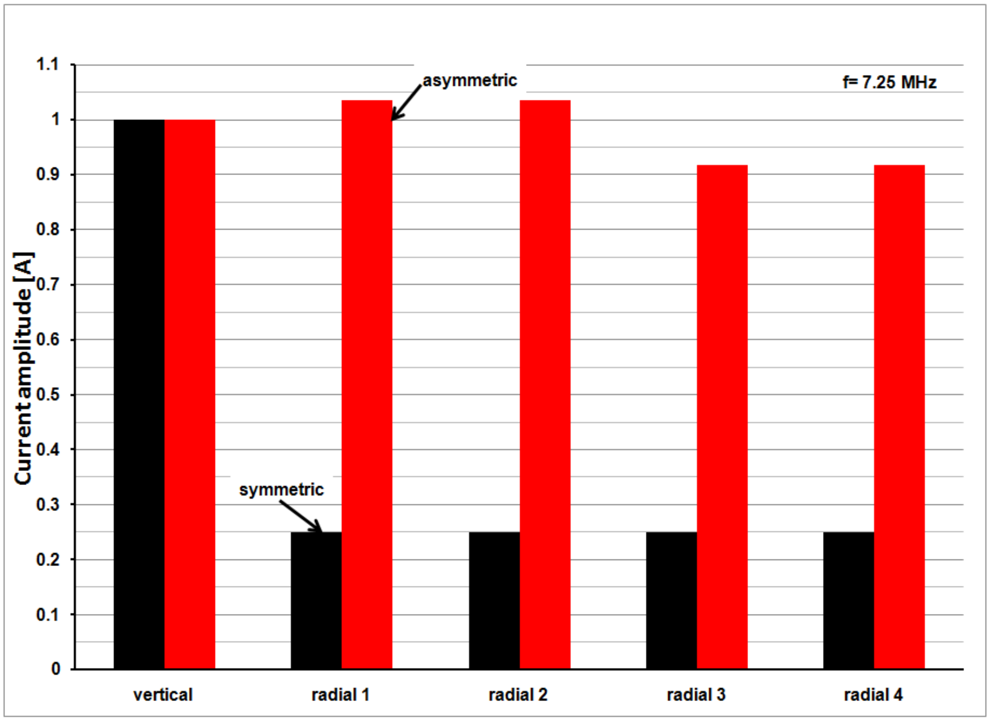

Figure 43 - Comparison of currents between symmetric and asymmetric radials.

The graph bars represent the current amplitudes at the base of the vertical and each of the radials immediately adjacent to the base of the vertical. The black bars are for symmetric radial lengths (L=M=34.1') and the red bars are for asymmetric radials (L=35.1' & M=33.1'). In the symmetric case each of the radials has an current of 0.25 A that sums to 1 A, the excitation current at the base of the vertical. The radial currents are also in phase with the base current.

With asymmetric radials the picture is very different: the current amplitudes are different between radial pairs 1 & 2 and 3 & 4 and the sum of the current amplitudes is <u>not</u> 1 A (the base current amplitude), it is much larger! This would seem to violate Kirchhoff's current law that requires the sum of the currents at a node to be zero. In this case the radial currents in the two pairs of radials are not in phase with each other or the vertical base current. The current in radials 1 & 2 is shifted by -62° from the base current and the current in radials 3 & 4 is shifted by +89°. The base and radial currents sum <u>vectorially</u> to 0 however, that satisfies Kirchhoff's law. These large asymmetric currents go a long way towards explaining the increased ground loss and pattern distortion. Note that the current asymmetry shown in figure 43 is for f=7.25 MHz. As the frequency is changed the pattern for the asymmetric currents in figure 43 will change a way similar to Weber's data shown in figure 38.

If we take the example of L=35.6' and M=33.1' and add a wire from the junction of the radials to a ground stake the Ga drops another -0.5 dB and the radial current asymmetry increases.

The forgoing examples represent only two particular cases. Obviously there are an infinite variety of radial fan distortions including radial lengths, azimuthal asymmetry, droop of the radials, etc, etc. As we increase the number of radials what we see is a rapid decrease in the sensitivity to asymmetric radial lengths. A primary effect of additional elevated radials (>4) is to reduce the sensitivity to radial asymmetry, nearby conductors, variations in ground conductivity or objects under the radial fan, and, as shown in Figure 27, more numerous radials reduce the potentials on the radials.

How can we tell if there is a problem in an existing radial fan? One way is to measure the current amplitudes in the individual radials close to the base of the vertical[1]. If the current amplitudes are significantly different between the radials and/or if the sum of the current amplitudes in the radials is greater than the base current then you have a problem. Current amplitude measurements can be made with a RF ammeter. More accurate measurements that also show the phase can be made using current transformers and an oscilloscope or a vector network analyzer[1].

FINAL COMMENTS

This discussion has shown that a vertical with an elevated ground system has many subtleties and many potentially useful variations but it has also shown that you cannot simply throw up a vertical with a few radials dangling in various directions and expect it to work properly. You have to take some care.

Are there a few simple rules which will keep us out of trouble? Here's my advice:

1) Use at least 10 to 12 radials.

2) Make an effort to have the radial system as symmetric as possible.

3) Keep the radial system as far as possible from other conductive objects.

4) While it is certainly possible to use almost any height for the vertical, I suggest you start with H= λ_o and trim the radials for resonance. This makes H a little tall shortens your radials (especially if you're using 10-12) and raise the feedpoint impedance a little.

5) Use a balun or common mode choke on the feedline **a**t the base of the vertical.

6) If you have a special problem situation by all means model some trial solutions first. That will save you a lot of time over cut-and-try in the field. If you can't afford NEC4 software, the free NEC2 software[41] will still be very helpful.

This article has covered a lot of ground looking in detail at the behavior of verticals with elevated ground systems. Despite the length of this article, it really just scratches the surface of the subject. There are many other topics that deserve attention. For example: a more detailed look at counterpoises or, in an array, the interaction between the radial systems associated with the individual verticals, the effect of non-level terrain, etc. I particularly recommend the articles by Al Christman, K3LC[18-33] that address many of these issues. While I hope the work reported here is helpful there is still lots more to be done before we can claim to really understand this class of antennas.

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